A Field Deformation Approach to Spatio-Temporal Forecast Verification of Gridded Sets

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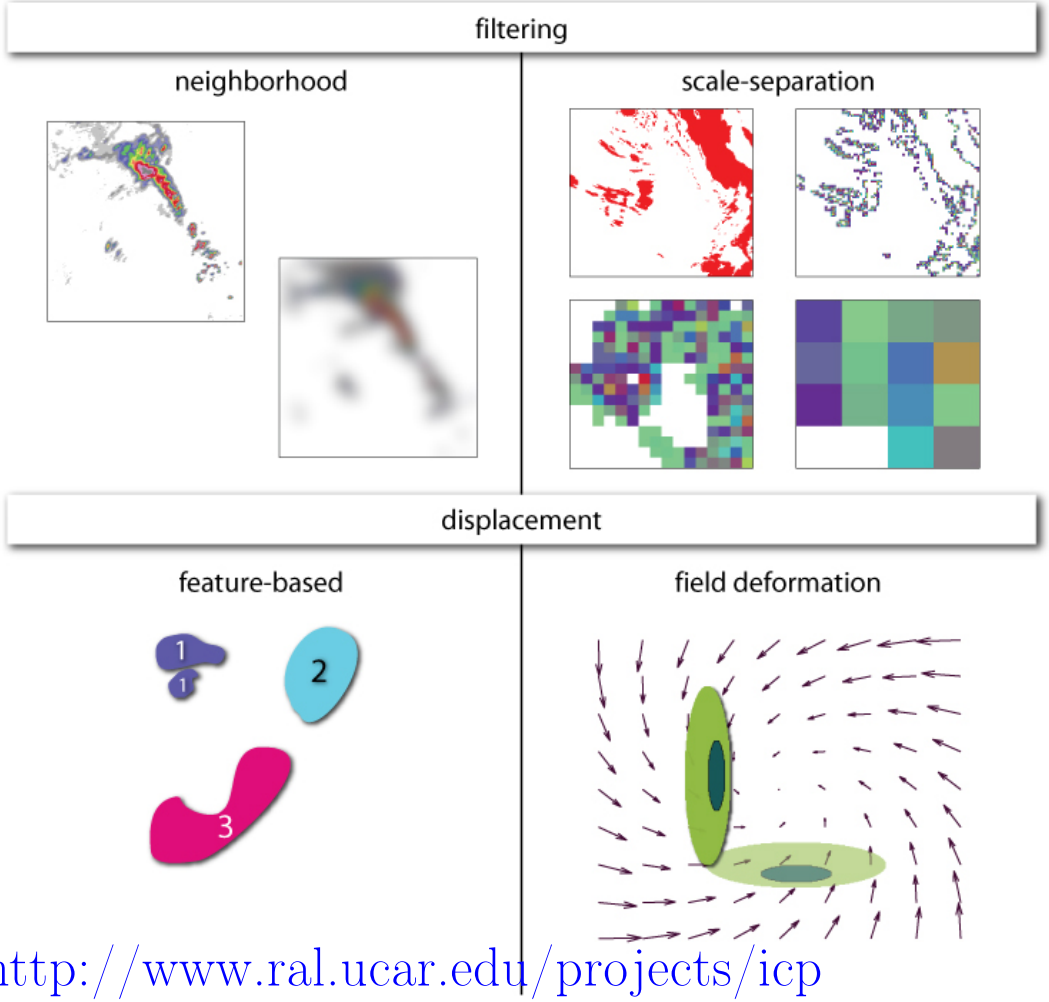
Spatial Forecast Verification Methods

Inter-Comparison Project (ICP)



http://www.ral.ucar.edu/projects/icp

Displacement Methods: Field deformation

Goal: Inform about how well the forecast captures spatial extent/patterns.

Examples:

Binary Image Metrics (Venugopal *et al.*, 2005; G. 2011;
Schwedler and Baldwin, 2011;
Zhu *et al.*, Submitted)

Optical Flow (e.g., Keil and Craig, 2008, 2009)

Image Warping (e.g., Alexander *et al.*, 1998;
G., Lindström and Lindgren, 2010)

Distortion representation (e.g., Hoffman *et al.*, 1995)

Field Deformation Methods: Image Warping

$$O(x, y) = F(W_x(x, y), W_y(x, y)) + \varepsilon$$

Field Deformation Methods: Image Warping

$$O(x, y) = F(W_x(x, y), W_y(x, y)) + \varepsilon$$

- W is a warping function that acts on both coordinates x and y of an image, and is applied to both coordinates;
- Many choices for W, e.g.,
  - polynomials (e.g., Alexander *et al.*, 1999; Dickinson and Brown, 1996)
  - B-splines (e.g., Engel *in prep*?)
  - Thin-plate splines (e.g., G., Lindström and Lindgren, 2010)
- Find optimal warp by optimizing a likelihood function.

Displacement Methods: Field Deformation

Field Deformation Methods: Image Warping

TPS warp function is a linear function in the 1-energy control points. That is,

$$\boldsymbol{W}(\boldsymbol{s}, \boldsymbol{p}^O, \boldsymbol{p}^F) = \boldsymbol{B}(\boldsymbol{s}, \boldsymbol{p}^O)\boldsymbol{p}^F$$



where B is a matrix of radial basis functions that is *pre-calculated*.

Field Deformation Methods: Image Warping

Optimize (log) likelihood:

$$\ell(\boldsymbol{p}^F|O,F,\boldsymbol{p}^O) = \log \mathrm{p}(O|F,\boldsymbol{p}^F,\boldsymbol{p}^O) + \log \mathrm{p}(\boldsymbol{p}^F|\boldsymbol{p}^O) + \log \mathrm{p}(\vartheta)$$

Intensity component

Location/spatial placement component

Possibly hyper-parameters

Field Deformation Methods: Image Warping

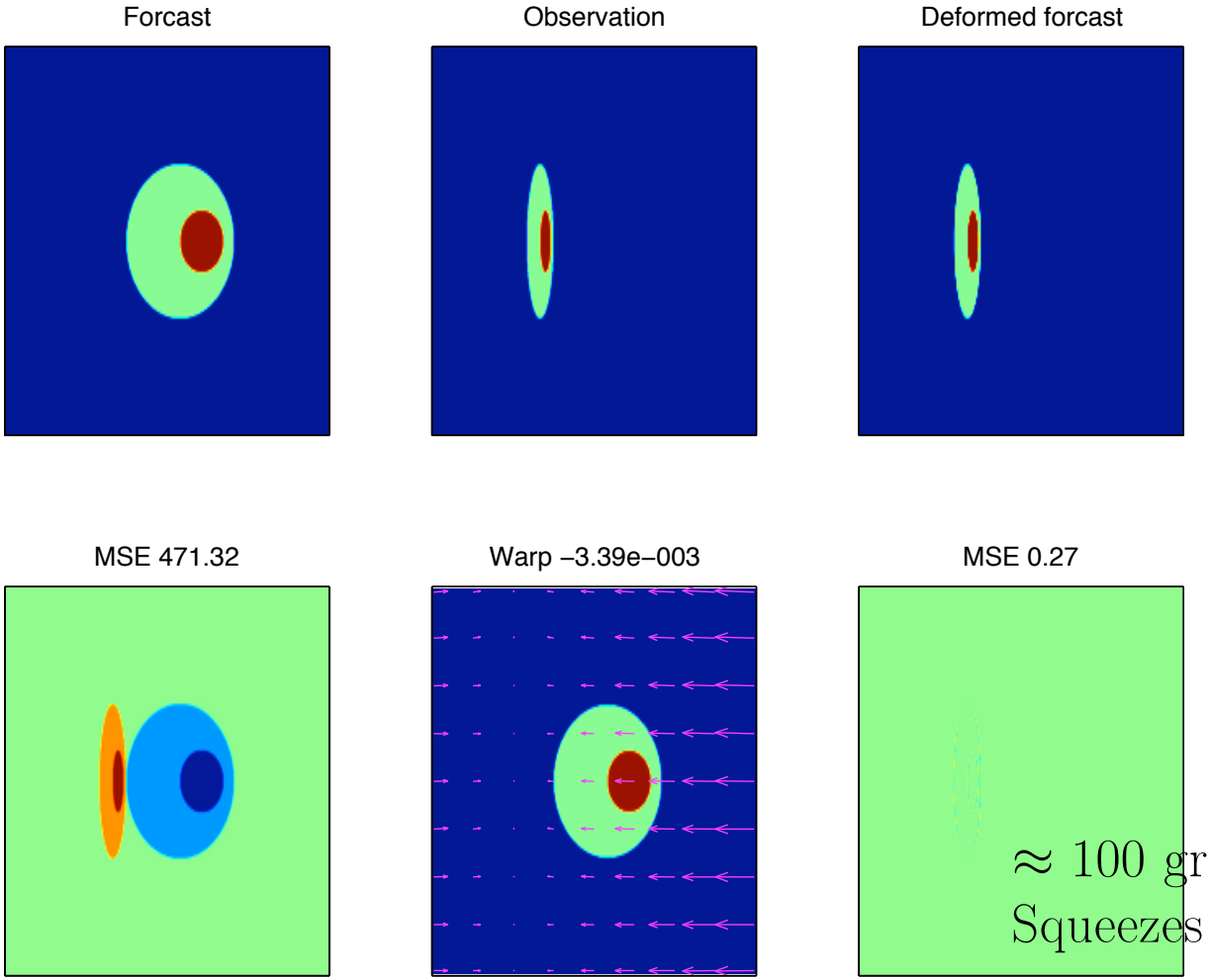
For the TPS Warp, the following optimization function can be used (assumes Gaussian errors, and a Markov Random Field Model for the control point differences).

$$Q(\boldsymbol{p}^F) = \frac{1}{2\sigma_\varepsilon^2} \sum \left(O(\boldsymbol{s}) - F(\boldsymbol{W}(\boldsymbol{s}))\right)^2 +$$

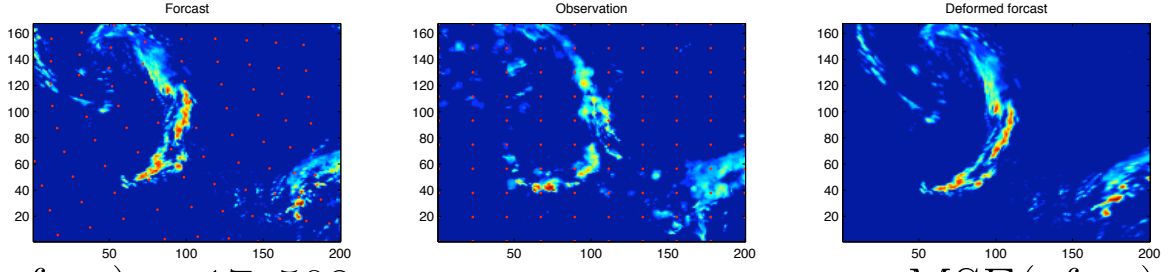
$$\frac{1}{2\sigma_\Delta^2} \left[(\boldsymbol{p}_x^F - \boldsymbol{p}_x^O)^T (\boldsymbol{I} - \boldsymbol{C})(\boldsymbol{p}_x^F - \boldsymbol{p}_x^O) + (\boldsymbol{p}_y^F - \boldsymbol{p}_y^O)^T (\boldsymbol{I} - \boldsymbol{C})(\boldsymbol{p}_y^F - \boldsymbol{p}_y^O)\right]$$

e.g., Åberg *et al.*, *Environmetrics*, **16**(8):833–848, 2005.

ICP Test Cases

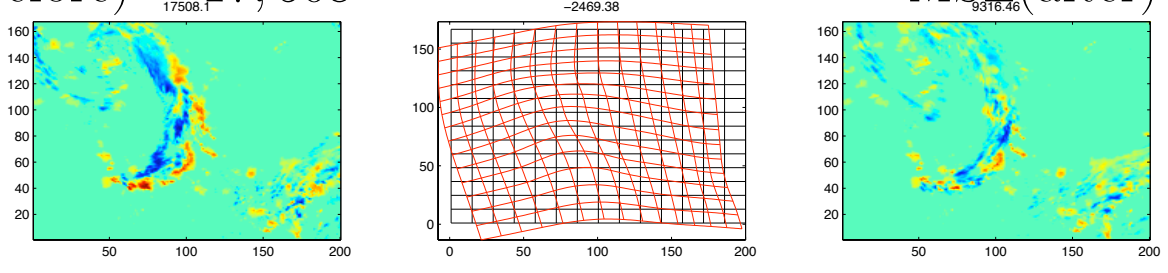


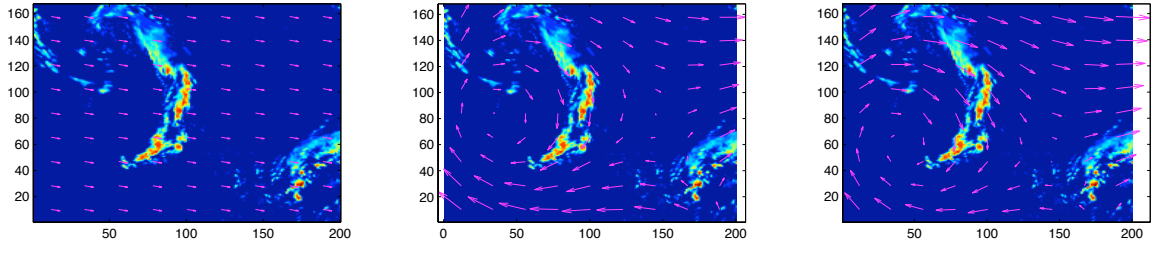
≈ 100 grid points west
Squeezes horizontally.



$\text{MSE}(\text{before}) = 17,508$

$\text{MSE}(\text{after}) = 9,316$





$$\frac{17{,}508 - 9316}{17{,}508} \approx 47\%$$

Space-Time Image Warp

Industrial Mathematical and Statistical Modeling Workshop for Graduate Students

July 19-27, 2010
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Sponsored by
Statistical and Applied Mathematical Sciences Institute (SAMSI), RTP, NC

Center for Research in Scientific Computation, Raleigh, NC

G. *et al.*, 2011. Spatial Forecast Verification: Image Warping. NCAR Technical Note, TN-482+STR, 23pp.

Space-Time Image Warp

Can timing errors be distinguished from spatial displacement errors?

Extension of 2-d spatial warping to space-time

Equations about the same, but with the added dimension. Tri-harmonic basis functions instead of 2-d TPS radial basis functions.

Space-Time Image Warp

Can timing errors be distinguished from spatial displacement errors?

Extension of 2-d spatial warping to space-time

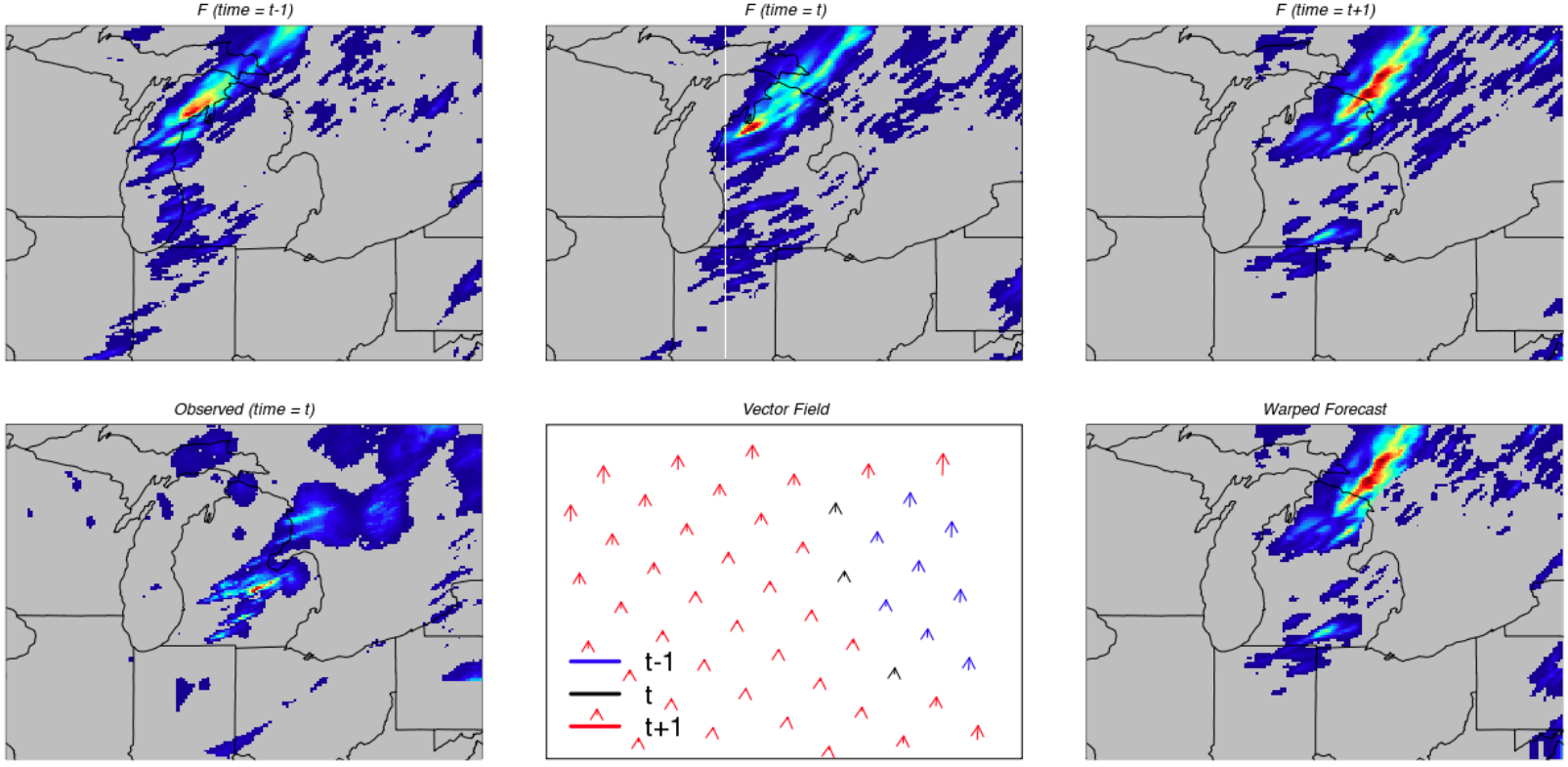
$$Q(\boldsymbol{p}^F) = \frac{1}{2\sigma_\varepsilon^2} \sum \left(O(\boldsymbol{s}) - F(\boldsymbol{W}(\boldsymbol{s}))\right)^2 +$$

$$\frac{1}{2\sigma_\Delta^2} \left[(\boldsymbol{p}_x^F - \boldsymbol{p}_x^O)^T(\boldsymbol{I} - \boldsymbol{C})(\boldsymbol{p}_x^F - \boldsymbol{p}_x^O) + (\boldsymbol{p}_y^F - \boldsymbol{p}_y^O)^T(\boldsymbol{I} - \boldsymbol{C})(\boldsymbol{p}_y^F - \boldsymbol{p}_y^O)\right] +$$

$$\frac{1}{\sigma_t^2} \left[(\boldsymbol{p}_t^F - \boldsymbol{p}_t^O)^T(\boldsymbol{I} - \boldsymbol{C})(\boldsymbol{p}_t^F - \boldsymbol{p}_t^O)\right]$$

Space-Time Image Warp

Example



Reduction in RMSE is over 50% after applying space-time warp. Most errors were spatial only.

Final Remarks

http://www.ral.ucar.edu/projects/icp

- See ICP web page under *References* and *Special Collection* for full references from these slides.
- ICP2 starting up! Goal is to investigate precipitation and wind fields over more complex terrain.
- Participation in the ICP is encouraged. Sign up to receive emails at the web site.
- New R software package for image warping is under development.
- New R Software package `SpatialVx` will contain all of the image warping techniques given here (via the to-be-submitted warping package), and most other techniques of the ICP, as well as others.