Predicting and evaluating extreme weather events

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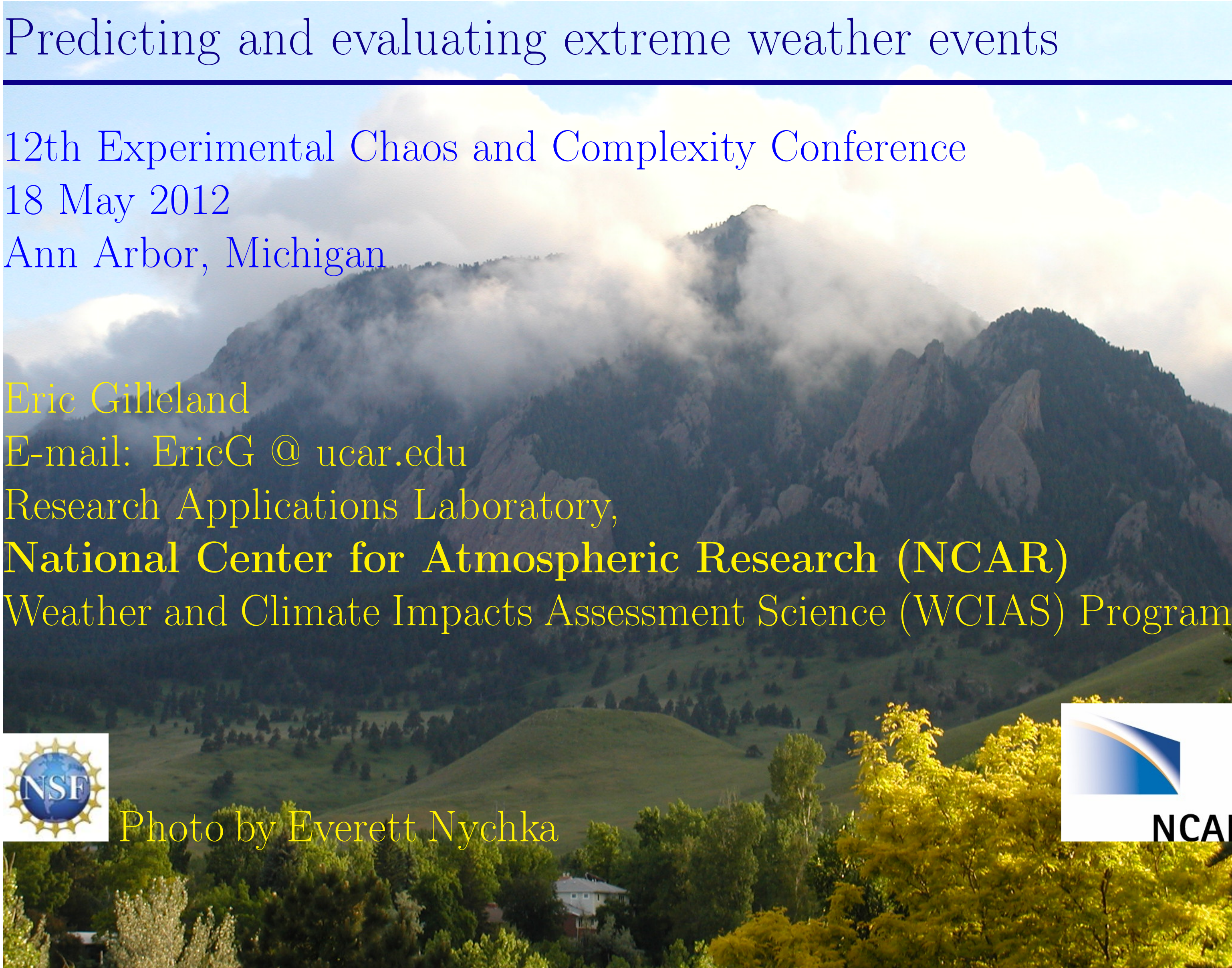

Photo by Everett Nychka

Motivation: Defining Extreme Events

POWER PLAY PAYOUT TABLE

MATCH	PRIZE	X2	X3	X4	X5
○○○○○●	Jackpot	POWER PLAY does not apply.			
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$$\Pr\{\text{Winning } \geq \$10,000 \text{ in one drawing}\} \approx 0.000001306024$$

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Hereafter, simplify the problem by ignoring the fact we can win small amounts, etc.

Motivation

Colorado Lottery

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In ten years, playing one ticket everyday, $\Pr\{\text{Winning } \geq \$10,000\} \approx$ 0.004793062

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In 100 years ≈ 0.05003321

In 1000 years ≈ 0.7686185

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Colorado Lottery

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In 100 years ≈ 0.05003321

In 1000 years ≈ 0.7686185

Law of small numbers: events with small probably rarely happen, but have many opportunities to happen. These follow a ***Poisson distribution***.

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Can also talk about waiting time probability. The ***exponential distribution*** models this. For example, the probability that it will take longer than a year to win the lottery (at one ticket per day) is ≈ 0.999523, longer than ten years ≈ 0.9952411, longer than 500 years ≈ 0.7877987, and so on (decays exponentially, but with a very slow rate).

Motivation

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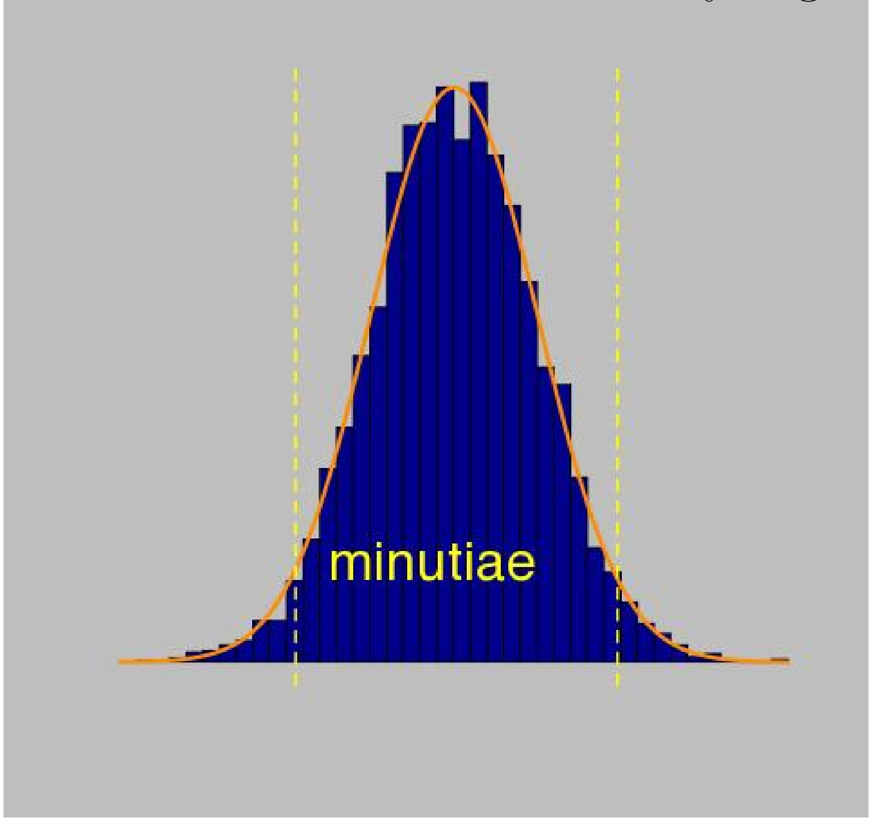
Another way to put it is that the expected number of years that it will take to win more than $10,000 in the lottery (buying one ticket per day) is about 2,096 years. If a ticket costs $1, then we can expect to spend $765,682.70 before winning at least $10,000.

Motivation

Law of Large Numbers, Sum Stability, Central Limit Theorem

And other results give theoretical support for use of the ***Normal distribution*** for analyzing most data.



Background: Extreme Value Theory (EVT)

Extremal Types Theorem

Theoretical support for using the **Extreme Value Distributions (EVD's)** for ***extrema***.

- Valid for maxima over very *large* blocks, or
- Excesses over a very *high* threshold.

It is possible that there is no valid distribution for extremes of a given random variable, but if one exists, it must be from the Generalized Extreme Value (GEV) family (block maxima) or the Generalized Pareto (GP) family (excesses over a high threshold). The two families are related.

Poisson process allows for a nice characterization of the threshold excess model that neatly ties it back to the GEV distribution.

Background

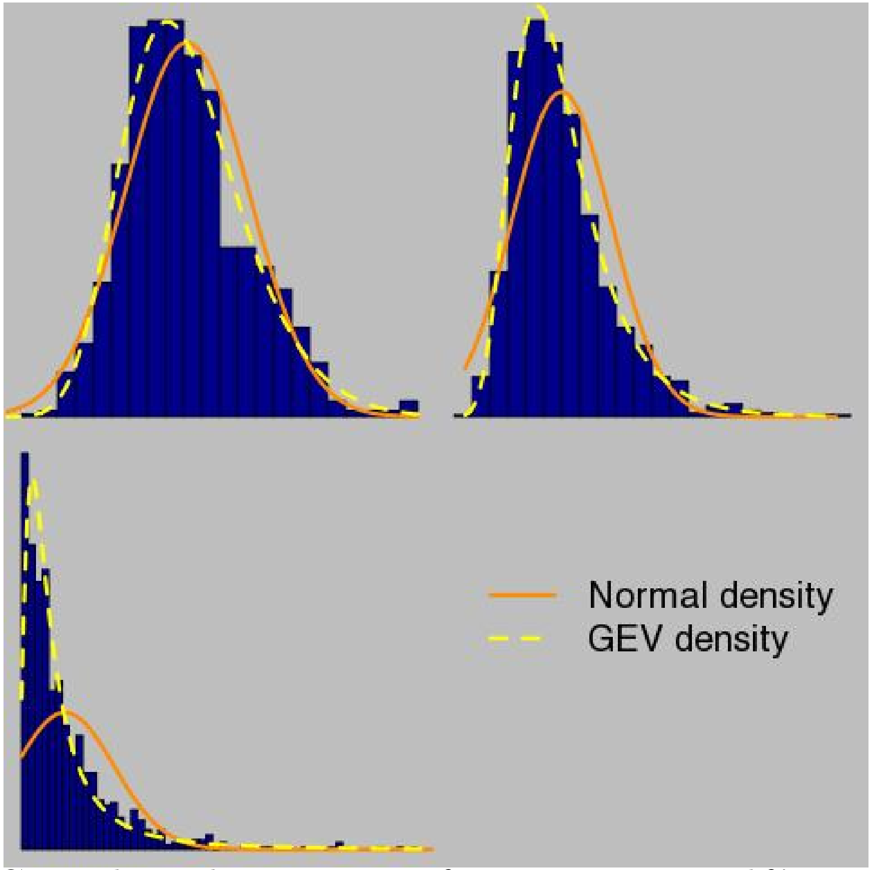
GEV

Three parameters: **location**, **scale** and **shape**.

$$\Pr\{M_n \leq z\} =$$

$$\exp\left\{-\left[1+\boldsymbol{\xi}\left(\frac{z-\boldsymbol{\mu}}{\boldsymbol{\sigma}}\right)\right]^{-1/\boldsymbol{\xi}}\right\}$$

$$M_n = \max\{X_1, \ldots, X_n\}$$



Simulated Maxima from various df's.

Background

GEV

Three parameters: **location**, **scale** and **shape**.

$$\Pr\{X \le z\} = \exp\left\{-\left[1+\boldsymbol{\xi}\left(\frac{z-\boldsymbol{\mu}}{\boldsymbol{\sigma}}\right)\right]^{-1/\boldsymbol{\xi}}\right\}$$

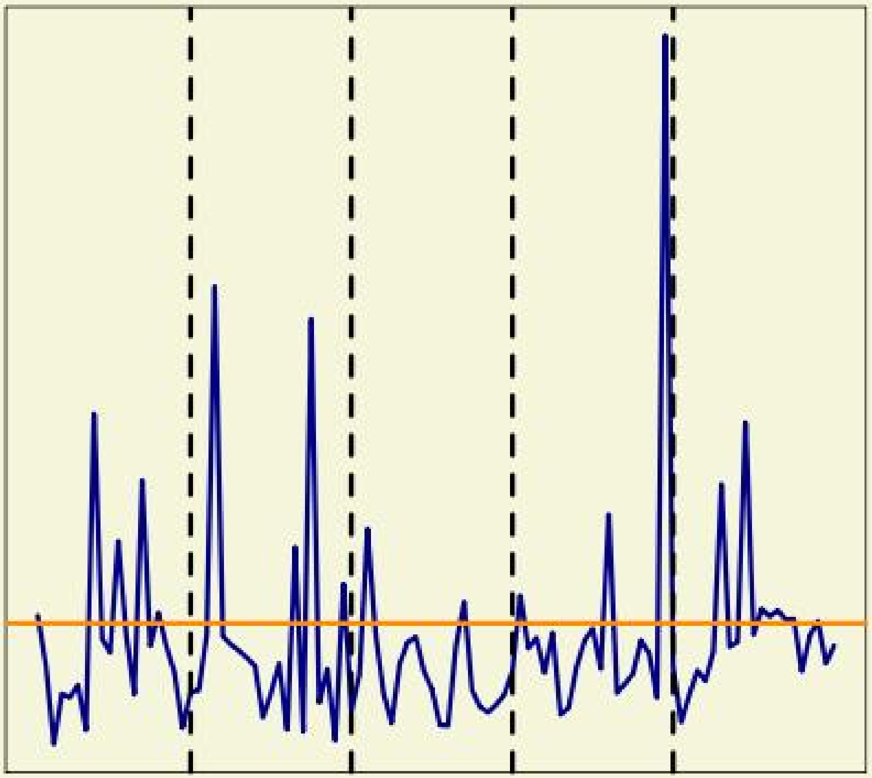
Three types of tail behavior:

1. Bounded upper tail ($\boldsymbol{\xi} < 0$, Weibull), Temperature, Wind Speed, Sea Level
2. light tail ($\boldsymbol{\xi} = 0$, Gumbel), and
3. heavy tail ($\boldsymbol{\xi} > 0$, Fréchet), Stream Flow, Precipitation, Economic Impacts.

Analogous situation for threshold excess approach, but focus is on the tail of these distributions.

Background

Block Maxima vs. POT



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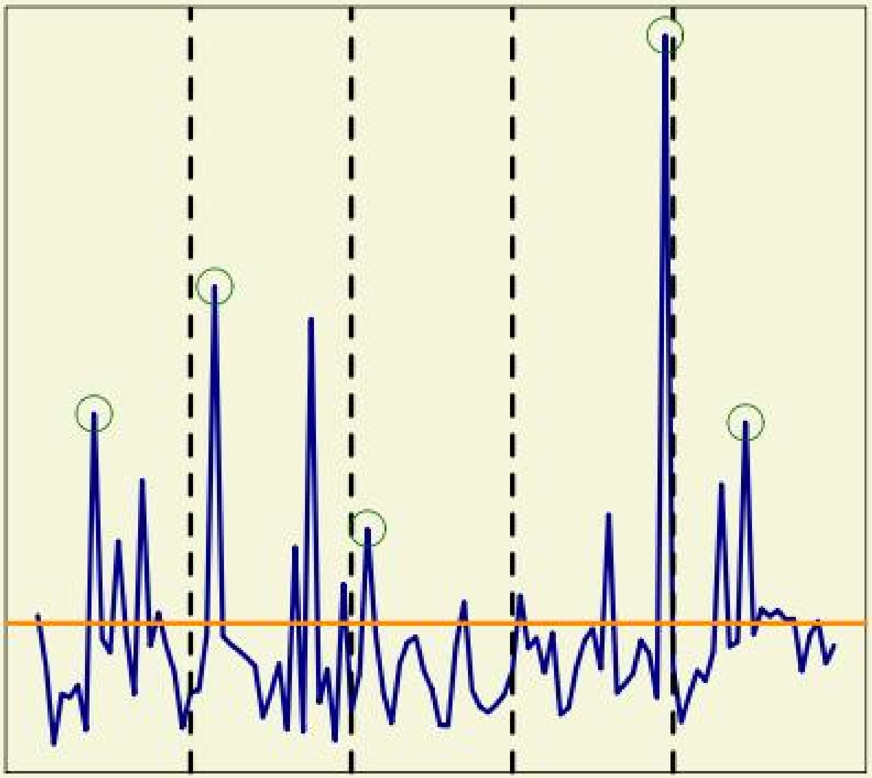
Block Maxima vs. POT



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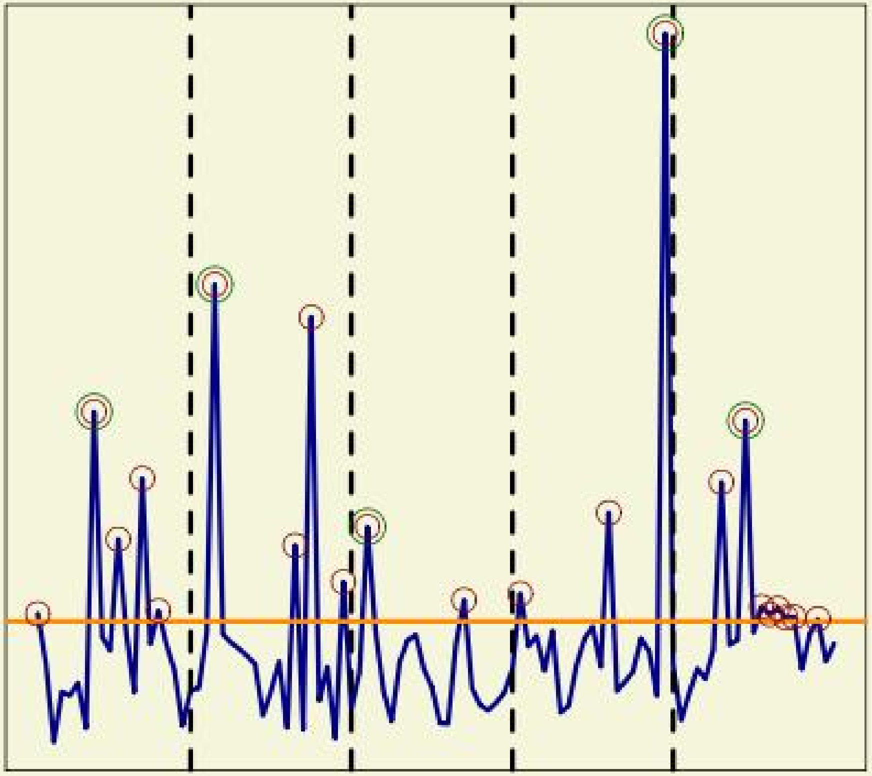
Block Maxima vs. POT



Example

Fort Collins, Colorado

daily precipitation amount

- Time series of daily precipitation amount (inches), 1900–1999.
- Semi-arid region.
- Marked annual cycle in precipitation (wettest in late spring/early summer, driest in winter).
- No obvious long-term trend.
- Recent flood, 28 July 1997. (substantial damage to Colorado State University)

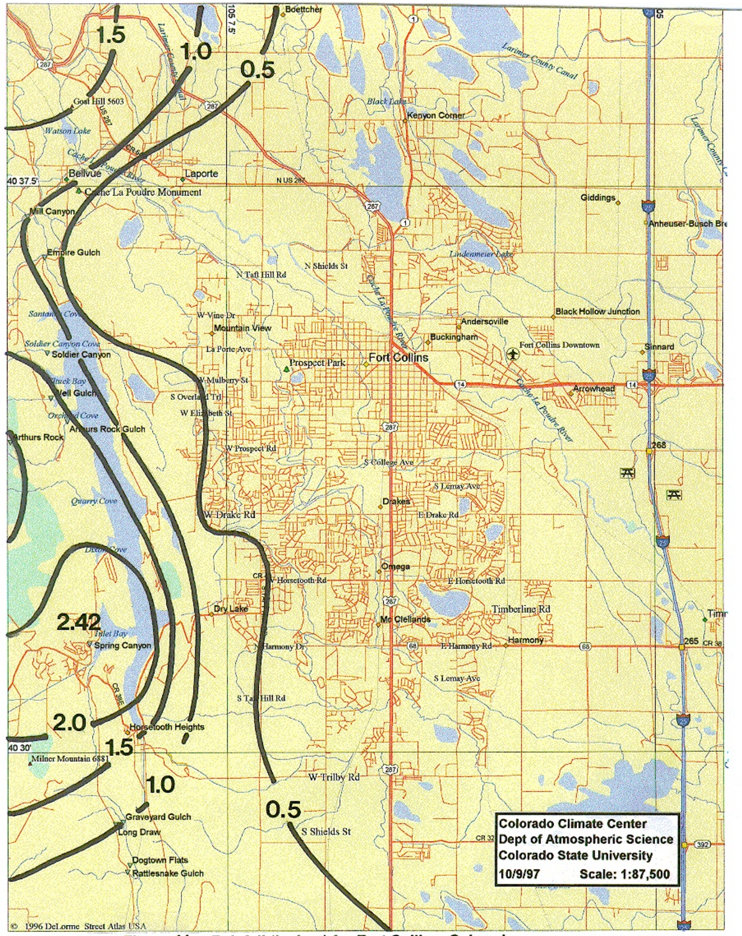
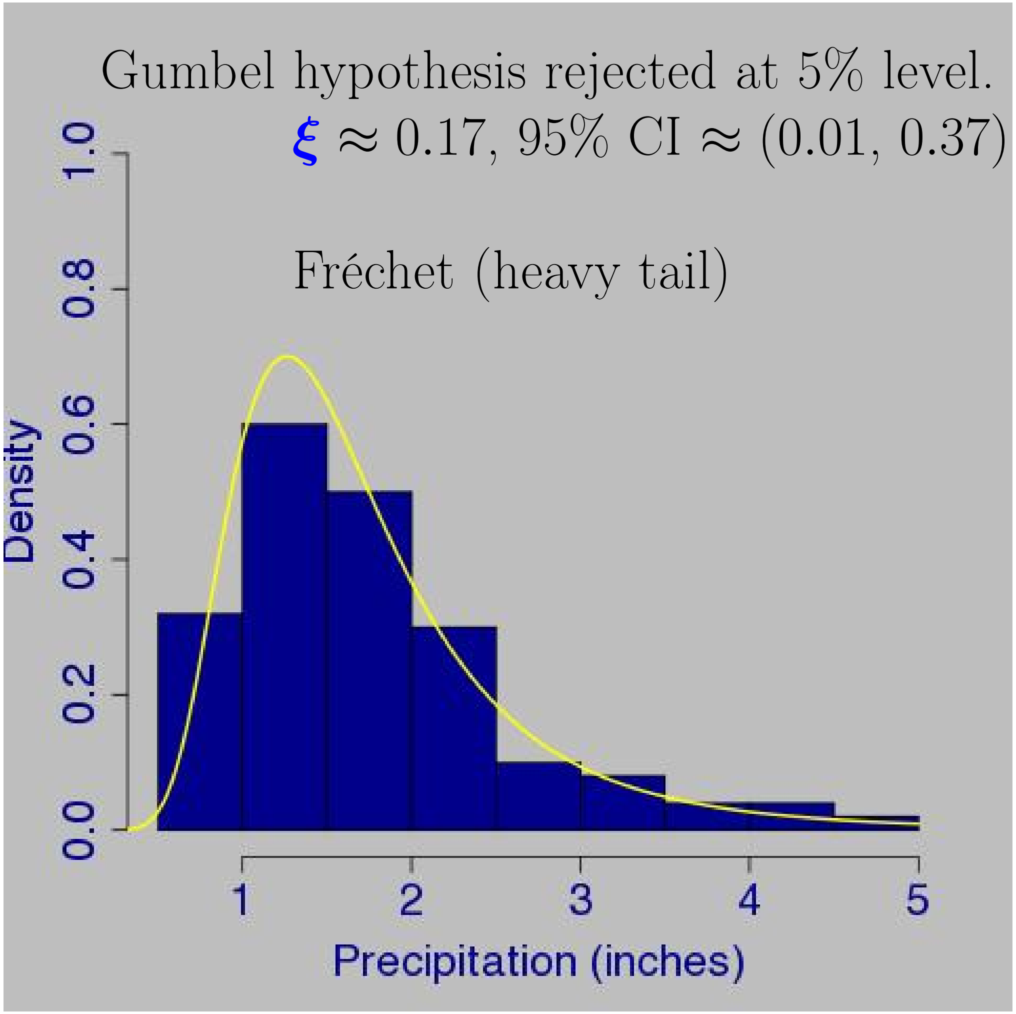


Figure 11. Rainfall (inches) for Fort Collins, Colorado, for 4:00-9:00 p.m. MDT for July 27, 1997

http://ccc.atmos.colostate.edu/~odie/rain.html

Examples

Fort Collins, Colorado precipitation

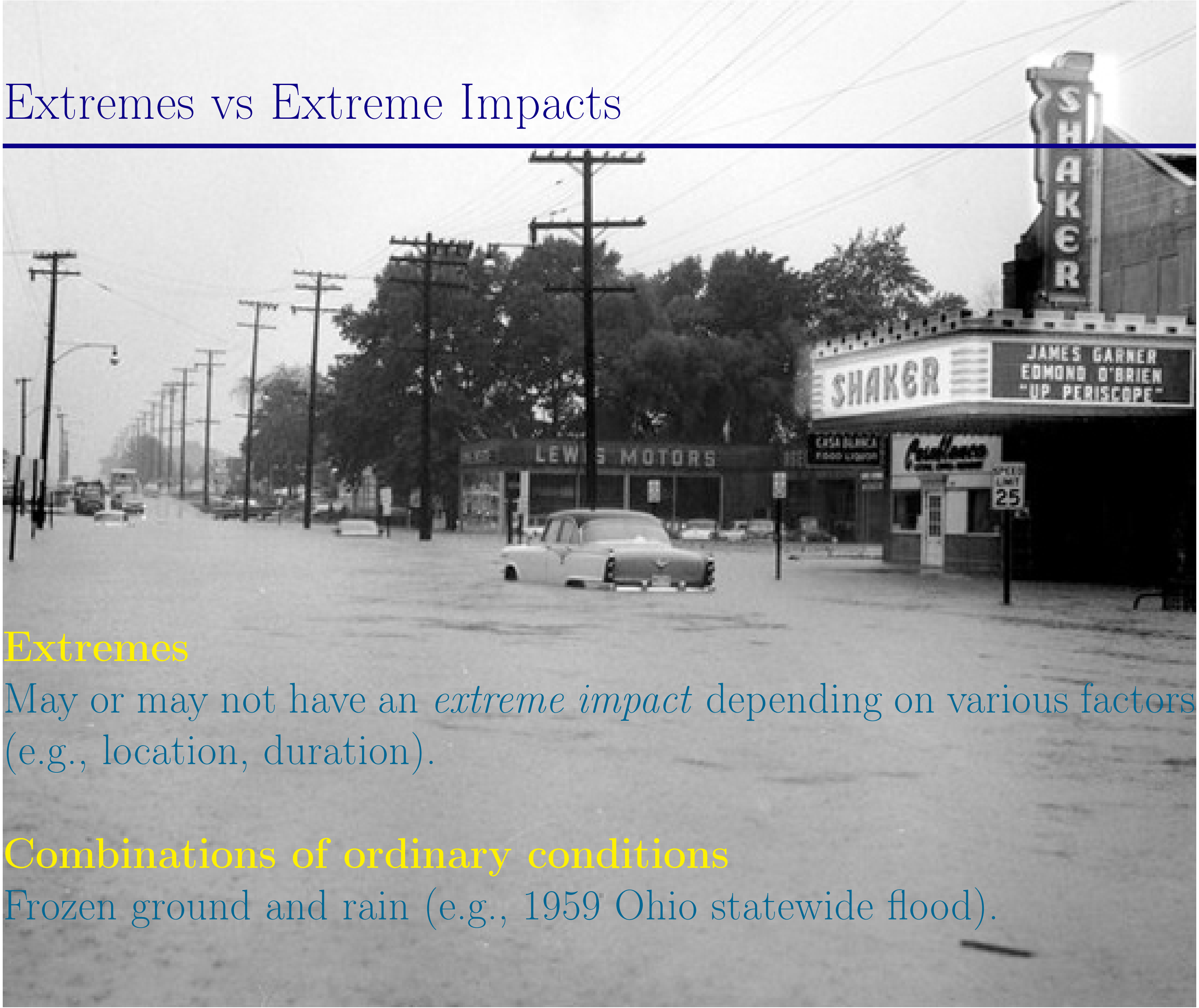


10-year return level ≈ 2.8 in.

$\Pr\{M_n \geq 3 \text{ in}\} \approx 0.08$

Return period for 3 in. is ≈ 12.5 years

Extremes vs Extreme Impacts



Extremes

May or may not have an *extreme impact* depending on various factors (e.g., location, duration).

Combinations of ordinary conditions

Frozen ground and rain (e.g., 1959 Ohio statewide flood).

Weather Spells: Many ways to define them technically

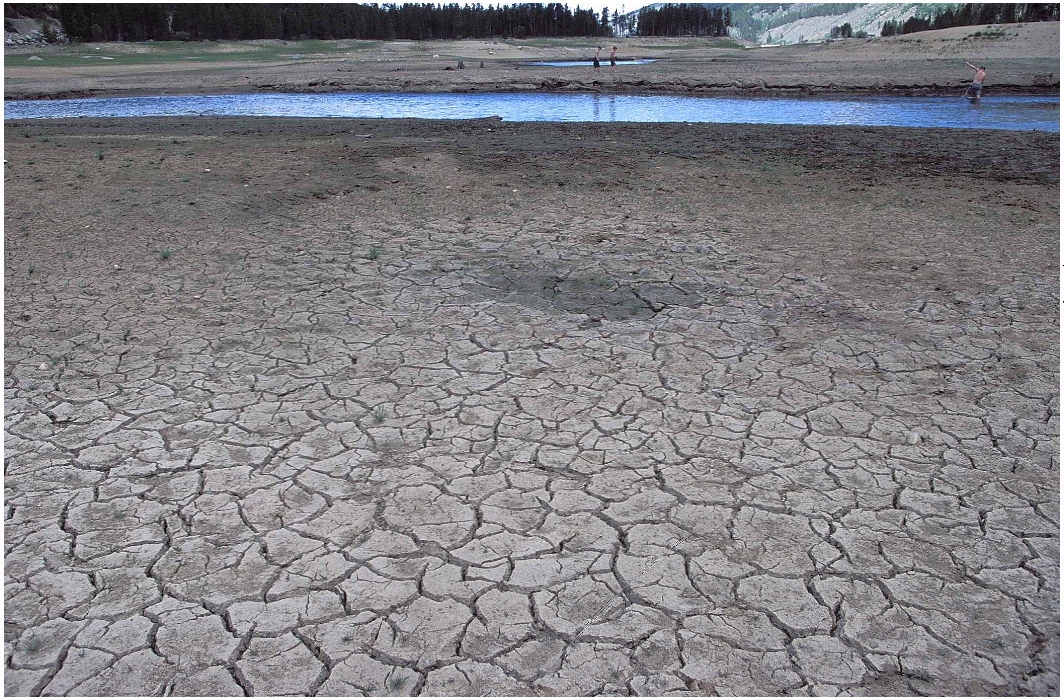


Photo from NCAR's digital image library, DIO1492

Do extremes of lengths of spells follow EV df's? (e.g., Cebrián and Abaurrea (2006), *J. Hydrometeorology*, **7**, 713–723, use a marked Poisson process approach)

The same type of weather spell may or may not be important depending on where it occurs.

What is a drought?

"a period of abnormally dry weather sufficiently prolonged for the lack of water to cause serious hydrologic imbalance in the affected area." -Glossary of Meteorology (1959)

Meteorological–a measure of departure of precipitation from normal. Due to climatic differences, what might be considered a drought in one location of the country may not be a drought in another location.

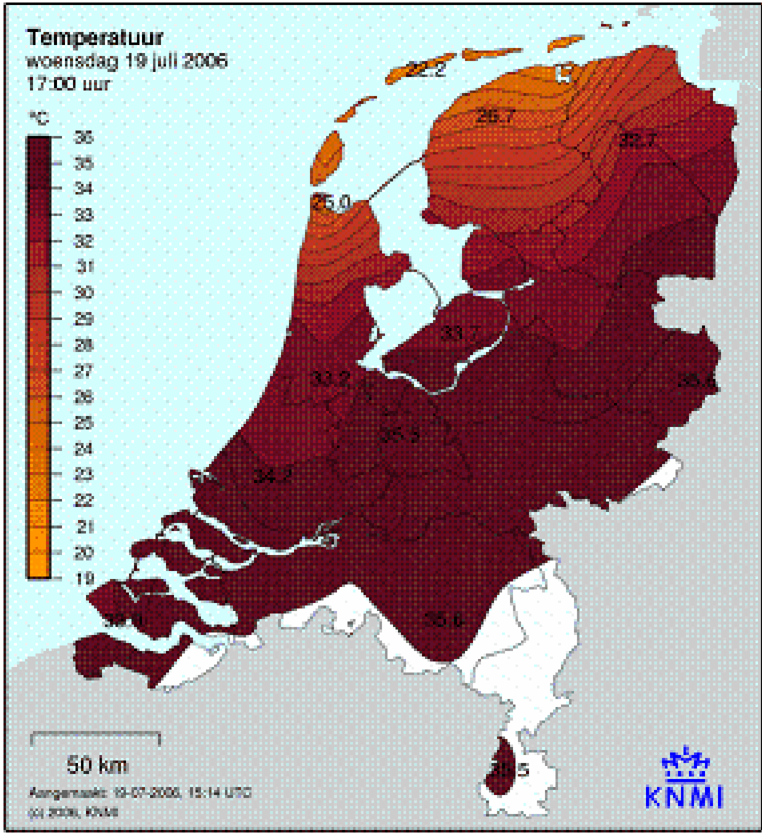
Agricultural–refers to a situation where the amount of moisture in the soil no longer meets the needs of a particular crop.

Hydrological–occurs when surface and subsurface water supplies are below normal.

Socioeconomic–refers to the situation that occurs when physical water shortages begin to affect people.

http://www.wrh.noaa.gov/fgz/science/drought.php?wfo=fgz

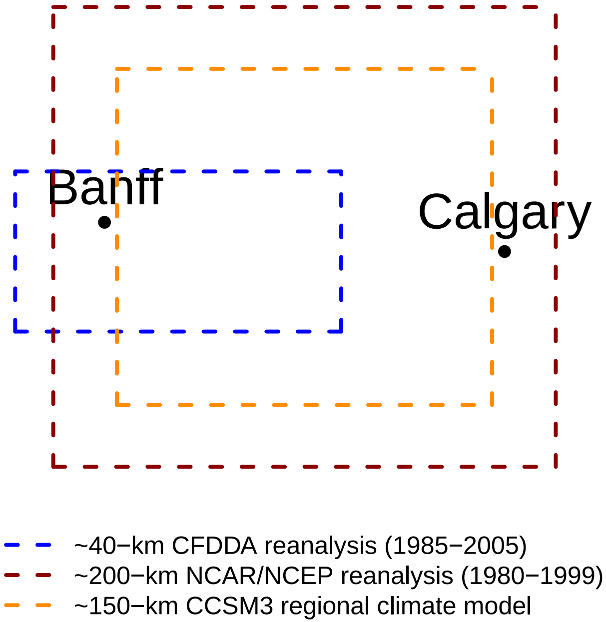
Scale of Extreme Atmospheric Events

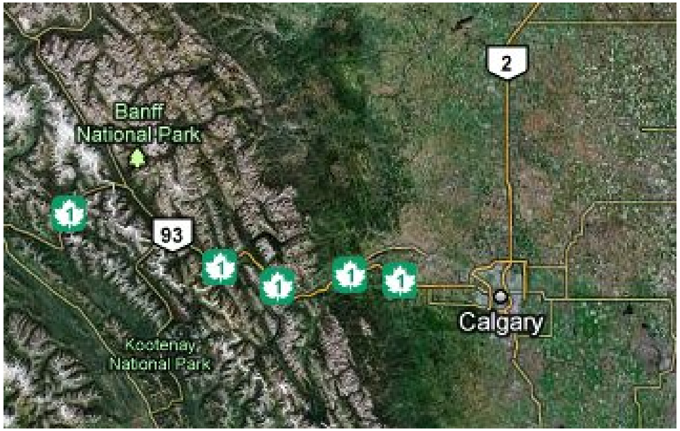


2006 European Heat Wave
(Fig. from KNMI)

F5 Tornado in Elie Manitoba
on Friday, June 22nd, 2007

Model/Reanalysis Resolution





Severe Weather

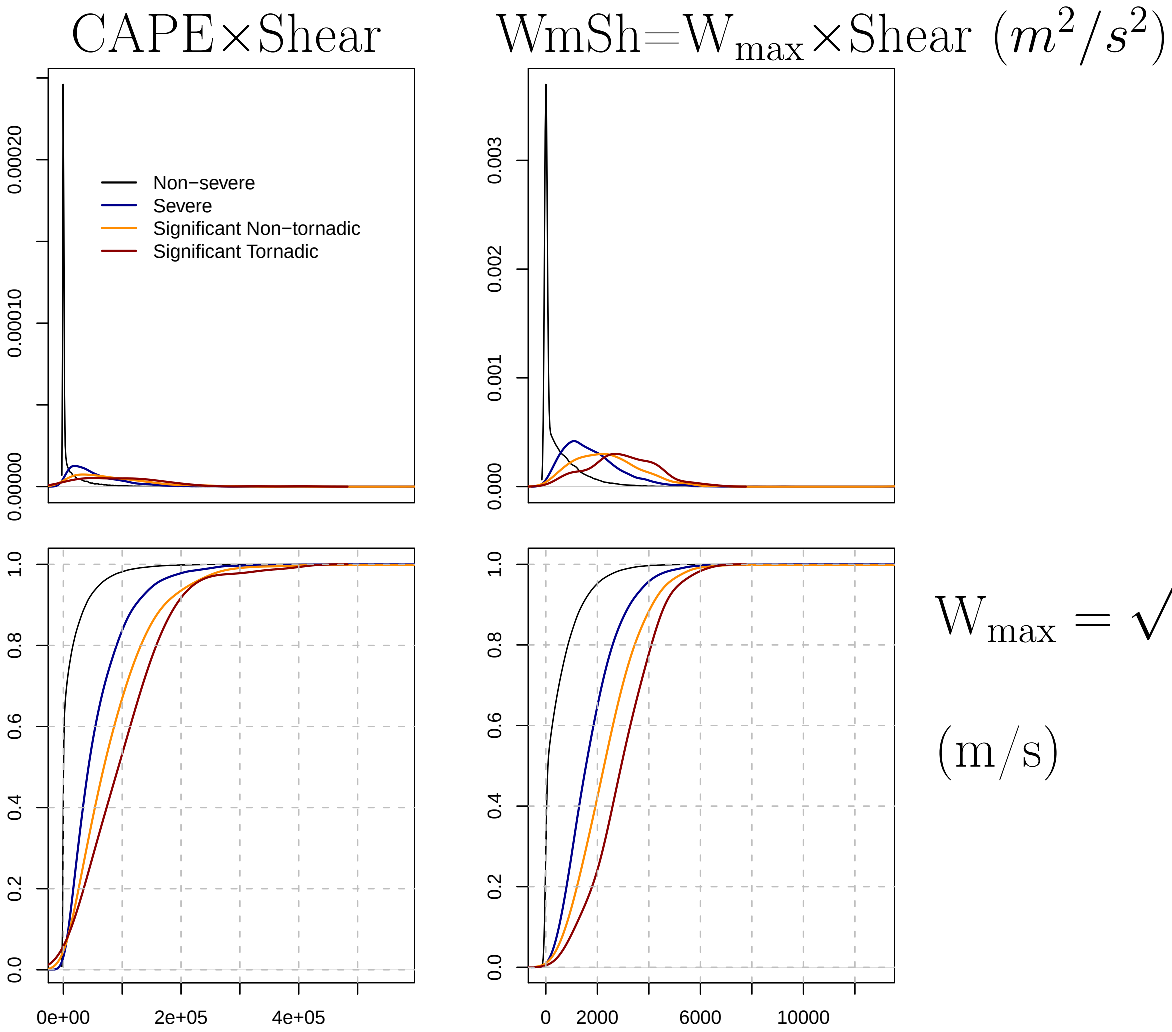
As model resolution increases, some severe weather phenomena, such as hurricanes, can be predicted. However, other types of severe weather may still require higher resolution.

- Use large-scale indicators to analyze conditions ripe for severe weather.
- Use climate models as drivers for finer scale weather models.
- Statistical approach to current trends in observations.
- Other?

Large-scale indicators for Severe weather

Non-severe	hail < 1.9 cm. $(3/4$ in.$)$ diameter winds < 55 kts. no tornado
Severe	Hail ≥ 1.9 cm. diameter winds ≤ 55 kts. and < 65 kts. or tornado
Significant Non-tornadic	Hail ≥ 5.07 cm. $(2$ in.$)$ diameter Winds ≥ 65 kts.
Significant Tornadic	Same as sig. tornadic with F2 (or greater) tornado.

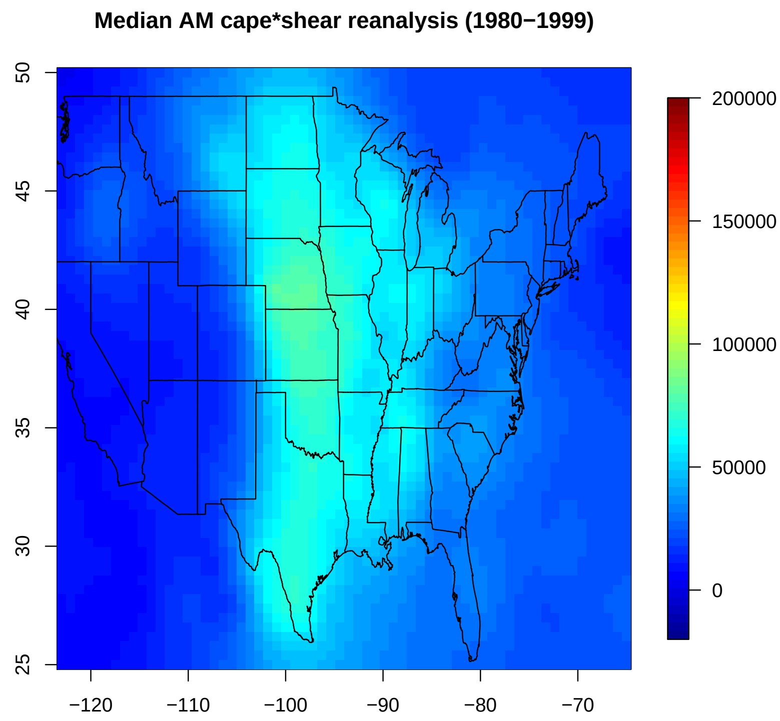
Large-scale indicators for Severe weather



$$W_{\max} = \sqrt{2 \cdot \mathrm{CAPE}}$$

(m/s)

CAPE (W_{max}) and 0-6 km shear data, or indeed, output

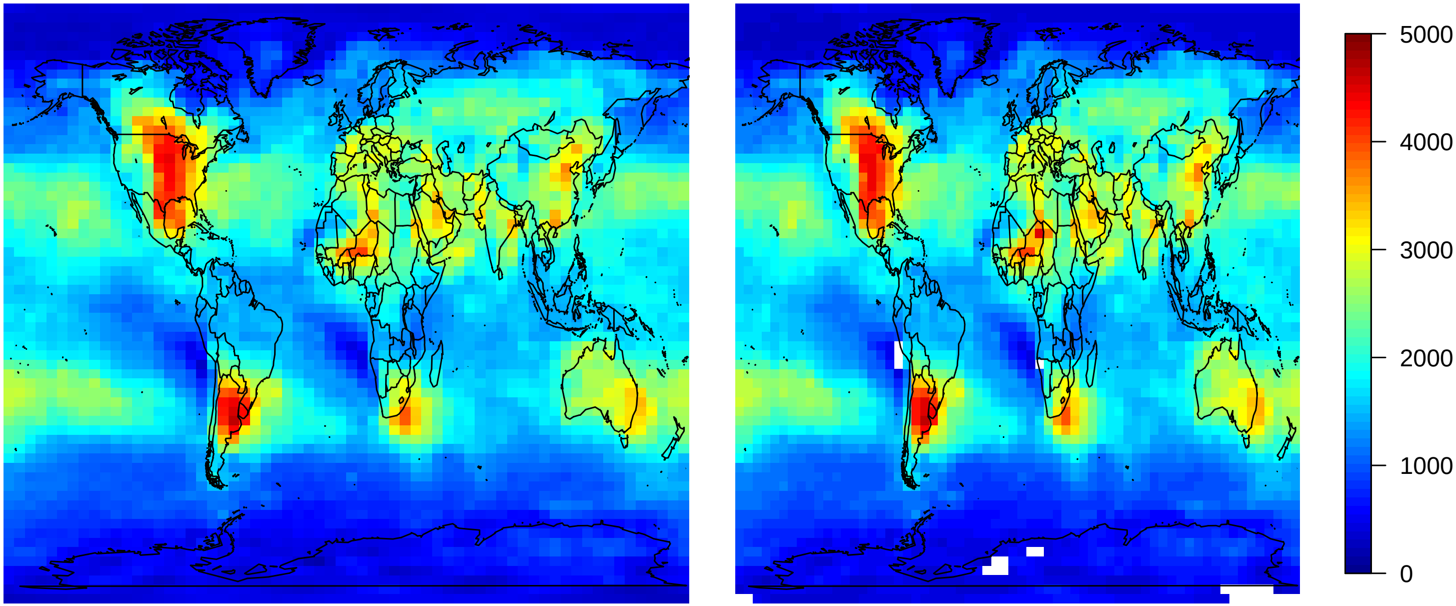


NCAR/NCEP global reanalysis: $1.875^o \times 1.915^o$ lon-lat grid, > 17K points, 6-hourly, 1958–1999. See: Brooks *et al.* (2003), *Atmos. Res.*, **67–68**, 73–94.

Large-scale indicators for Severe weather

WmSh=$W_{max} \times$ Shear

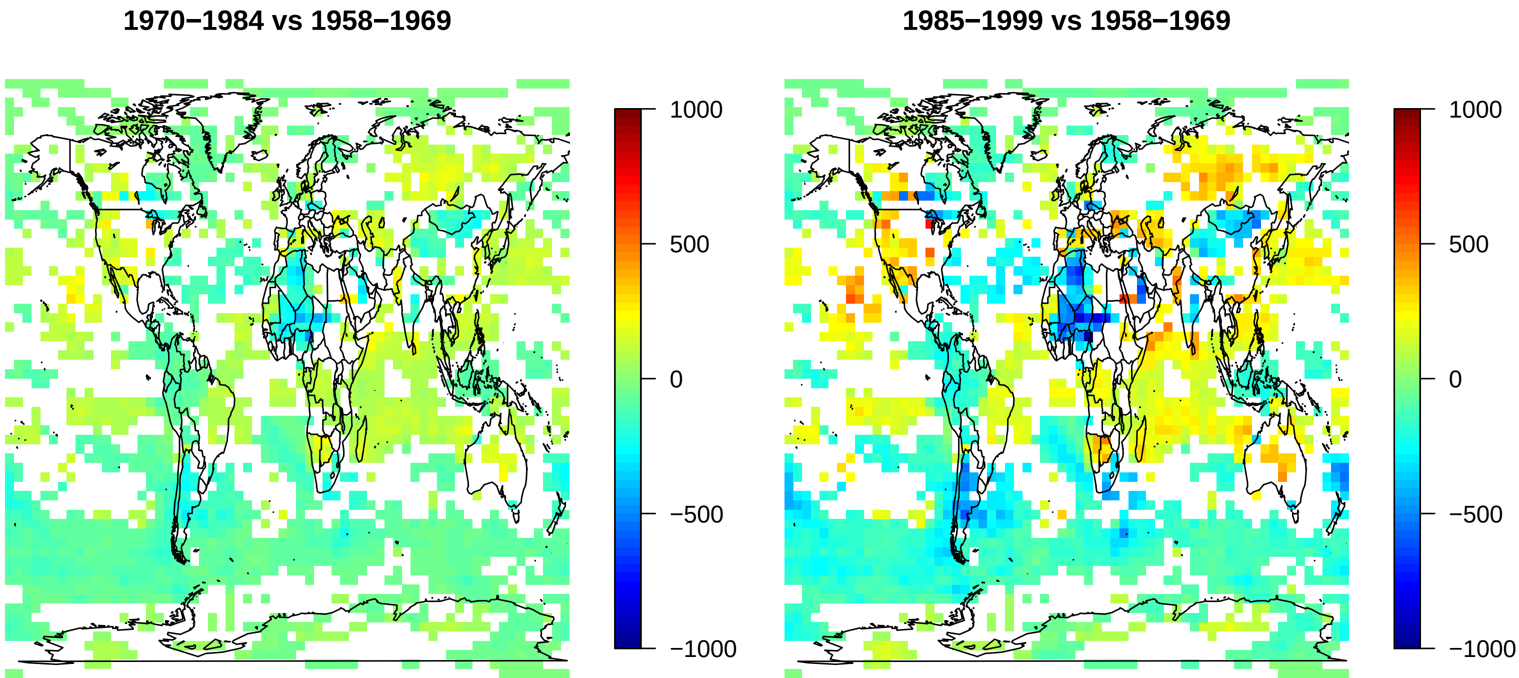
AM Reanalysis (95-th quantile) 20-yr GEV return level



Units are off here (shear is in knots instead of m/s, so take half of what you see).

Large-scale indicators for Severe weather

$\mathrm{GEV}(\mu(t) = \mu_0 + \mu_1 t, \sigma, \xi)$,
$t = 0$ (1958–1969), $t = 1$ (1970–1984), $t = 2$ (1985–1999).
20-year return levels (i.e., 95-th percentile, m/s).



Min. diff. from $t = 0$ to $t = 2$ is ≈ -750 m/s, max. is ≈ 400 m/s.
25th percentile of diff's is ≈ -100 m/s, 75th percentile is ≈ 75 m/s.

Large-scale indicators for Severe weather

Threshold excess modeling using Bayesian Hierarchical Models (BHM)

Industrial Mathematical and Statistical Modeling (IMSM) Workshop for Graduate Students. Center for Research in Scientific Computation, Raleigh, North Carolina and the Statistical and Applied Mathematical Sciences Institute (SAMSI), Research Triangle Park, North Carolina, 20-28 July 2009.

Paper in *Environmetrics*, **22**, 294–303:

Heaton, M.J., M. Katzfuss, S. Ramachandar, K. Pedings, Y. Li, E. Gilleland, E. Mannshardt-Shamseldin, and R.L. Smith, 2009. Spatio-temporal models for extreme weather using large-scale indicators.

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Large-scale indicators for Severe weather

Three models of increasing complexity applied to threshold excesses (over the 95-th quantile of *daily* maximum WmSh).

Model 1: Very Simple

GPD with (ln) scale and shape parameter varying by region only.

$y_{td}(\boldsymbol{s}_l)|\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l) \overset{\text{iid}}{\sim} \text{GPD}(\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l))$, where

$$\ln(\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l)) = \alpha_\psi + \mathbf{1}_{\boldsymbol{s}_l \in \mathcal{A}_\psi}\delta_\psi,$$

and

$$\xi(\boldsymbol{s}_l) = \alpha_\xi + \mathbf{1}_{\boldsymbol{s}_l \in \mathcal{A}_\xi}\delta_\xi,$$

with $\mathcal{A}_x$ somewhat arbitrarily chosen regions representing areas of exceptional values of these parameters as estimated via MLE at individual locations (this roughly translates to the "*tornado alley*"). Priors for these parameters are taken as $\alpha_\psi \sim N(5.5, 1)$, $\delta_\psi \sim N(0, 1)$, $\alpha_\xi \sim N(0, 0.2^2)$, $\delta_\xi \sim N(0, 0.2^2)$.

Large-scale indicators for Severe weather

Model 2:

GPD with Gaussian process for the (ln) scale parameter, and shape parameter varies according to region.

$y_{td}(\boldsymbol{s}_l)|\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l) \overset{\text{iid}}{\sim} \text{GPD}(\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l))$, where

$$\ln(\psi_{u(\boldsymbol{s}_l)}(\boldsymbol{s}_l)) \sim GP((\mu_\psi, \tau_\psi^2, \phi_\psi),$$

and

$$\xi(\boldsymbol{s}_l) = \alpha_\xi + \mathbf{1}_{\boldsymbol{s}_l \in \mathcal{A}_\xi} \delta_\xi,$$

with $\text{Cov}(\ln(\psi(\boldsymbol{s}_l)), \ln(\psi(\boldsymbol{s}_k))) = \tau^2 \exp\{-\phi_\psi \|\boldsymbol{s}_l - \boldsymbol{s}_k\|\}$, and $\|\cdot\|$ the spherical distance in miles. Priors are the same as model 2, with additional priors for $\mu_\psi \sim \text{Unif}(-\infty, \infty)$, $\tau_\psi^2 \sim \text{IG}(2.1, 3)$, and $\phi_\psi \sim \text{Unif}(0.001, 0.1)$.

Model 3:

Point Process with temporal trend for location parameter, trivariate Gaussian process for location and (ln) scale parameters, and shape parameter varying according to region as in other two models.

$$x_{td}(\boldsymbol{s}_l)|x_{td}(\boldsymbol{s}_l) > u(\boldsymbol{s}_l), \beta_0(\boldsymbol{s}_l), \beta_1(\boldsymbol{s}_l), \sigma(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l) \overset{\text{iid}}{\sim}$$

$$\text{PP}(\beta_0(\boldsymbol{s}_l) + \beta_1(\boldsymbol{s}_l)t, \sigma(\boldsymbol{s}_l), \xi(\boldsymbol{s}_l)),$$

where

$$(\beta_0(\boldsymbol{s}_l), \beta_1(\boldsymbol{s}_l), \ln(\sigma(\boldsymbol{s}_l)))^T \sim \text{GP}_3(\boldsymbol{\mu}_{M_3}, \phi_{M_3}, \boldsymbol{\Gamma}),$$

and

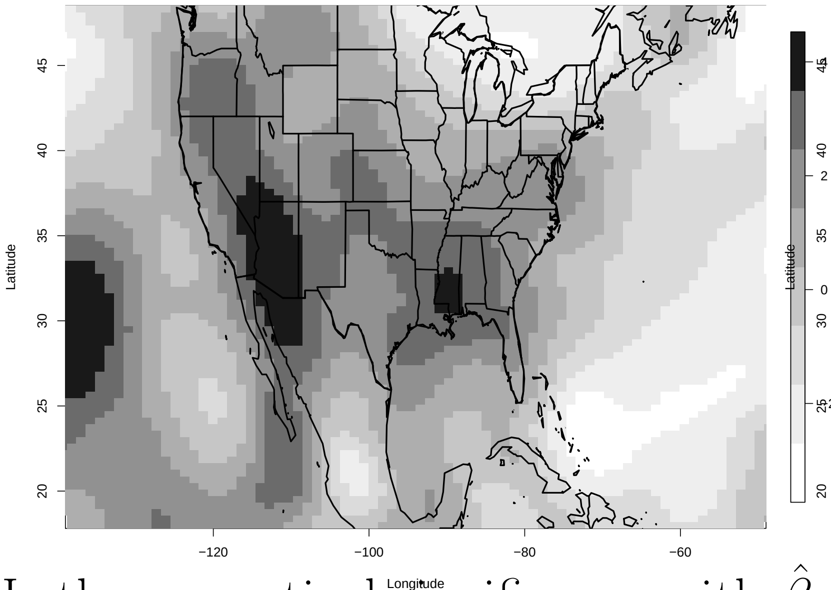
$$\xi(\boldsymbol{s}_l) = \alpha_\xi + \mathbf{1}_{\boldsymbol{s}_l \in \mathcal{A}_\xi} \delta_\xi,$$

with GP_3 a trivariate Gaussian process induced via coregionalization (Gelfand *et al.* 2004), $\boldsymbol{\mu}_{M_3} = (\mu_{\beta_0}, \mu_{\beta_1}, \mu_\sigma)^T$, $\phi_{M_3} = (\phi_1, \phi_2, \phi_3)^T$, and $\boldsymbol{\Gamma}$ is a 3×3 lower triangular matrix with entries γ_{ij}.

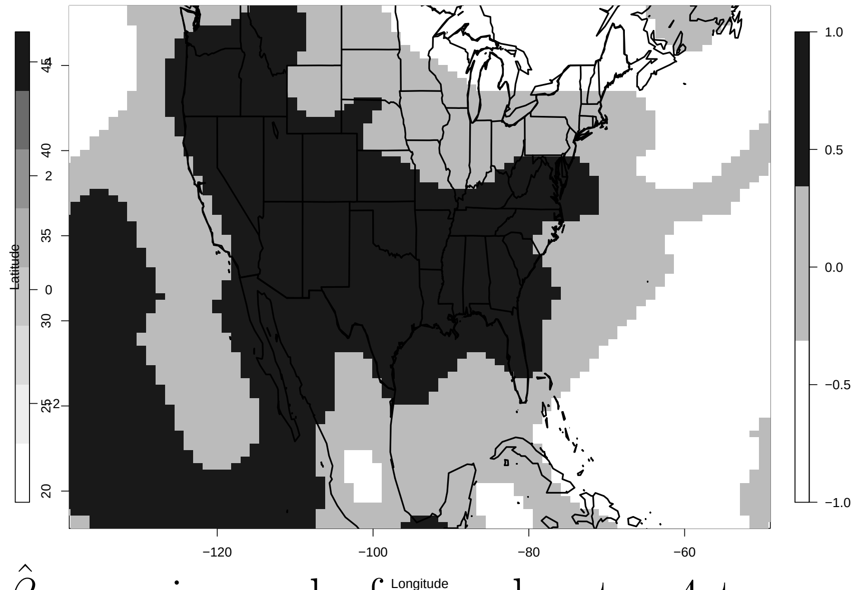
Large-scale indicators for Severe weather

Model 3 is best according to DIC (also most useful).

$\hat{\beta}_1$ Values



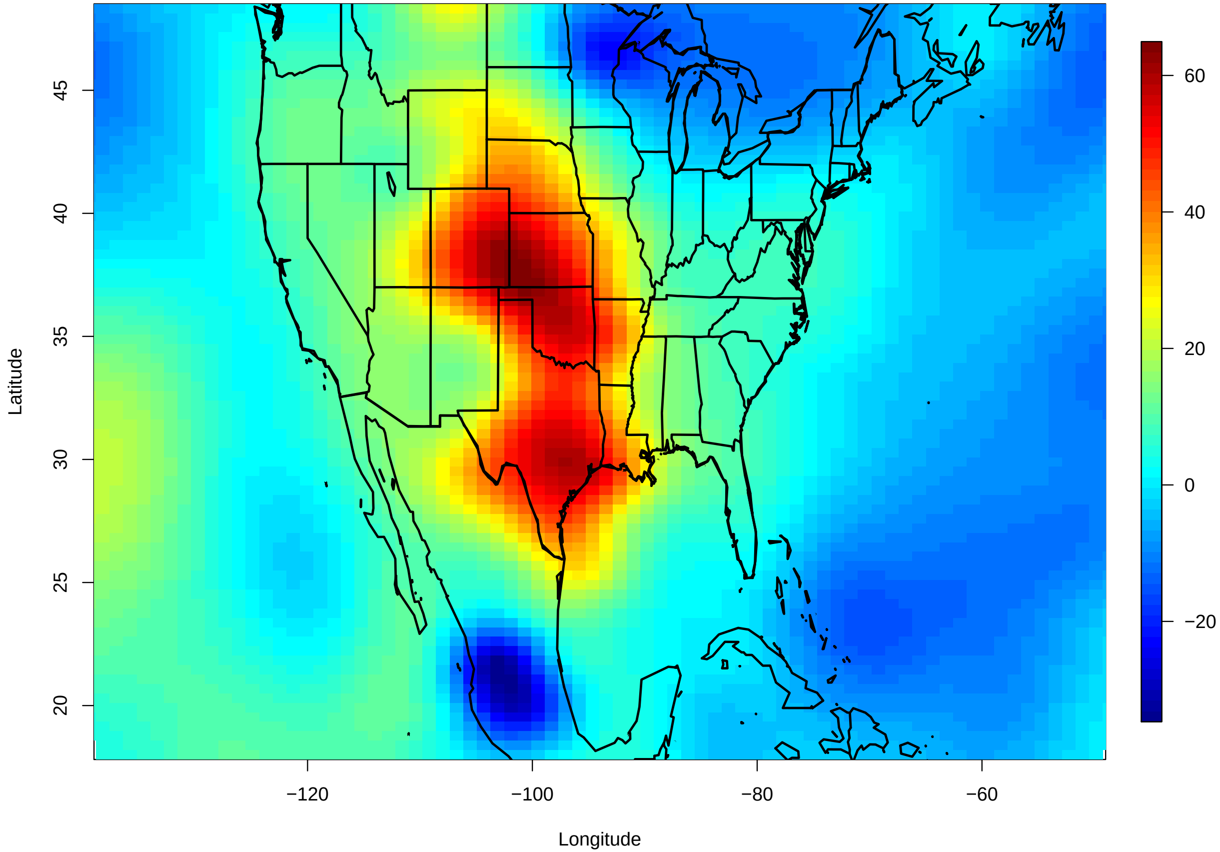
Statistical Significance



Is there practical significance with $\hat{\beta}_1$ ranging only from about -4 to 4 (e.g., 4×42 years is only 168 m/s difference *in location parameter*)?

Large-scale indicators for Severe weather

Twenty-year return level differences as calculated from the posterior means of Model 3 for 1999 vs 1958. Practical significance?



Large-scale indicators for Severe weather

Conditional EVA

Heffernan and Tawn (2004), *J. R. Statist. Soc. B*, **66**, 497–546.

- Allows as many variates as you like.
- Different assumptions than the usual multivariate EVA approach: condition on one variable's being large, and find the joint conditional distribution of other variables.
- Uncertainty obtained through bootstrapping (can be slow).
- Model for positively associated pairs of r.v.'s has a simple form.
- Semi-parametric model.
- Theoretical justification for extrapolating beyond the range of the sample.

Conditional EVA

For simplicity, take the bivariate case, with random variables X and Y.

1. Find marginal distributions, f_X and f_Y, using univariate EVT.
2. Transform X and Y to the Gumbel scale (w/o loss of generality).
3. Then, $y|X = x = \alpha x + x^{\beta} Z$, $Z \sim$ std. df., u a high threshold.

$$\hat{Z} = \frac{y|X = x - \hat{\alpha} x}{x^{\hat{\beta}}}.$$

   Estimate $\alpha \in [0, 1]$ and $\beta \in (-\infty, 1)$ using, e.g., nls.
4. Using $\hat{Z}$, characterize $f(\hat{Z})$ (e.g., kernel density, resampling).
5. Sample at random from $\{\hat{Z}_i\}_{i=1}^{n}$, calculate y (step 3), back-transform (step 1).

Large-scale indicators for Severe weather

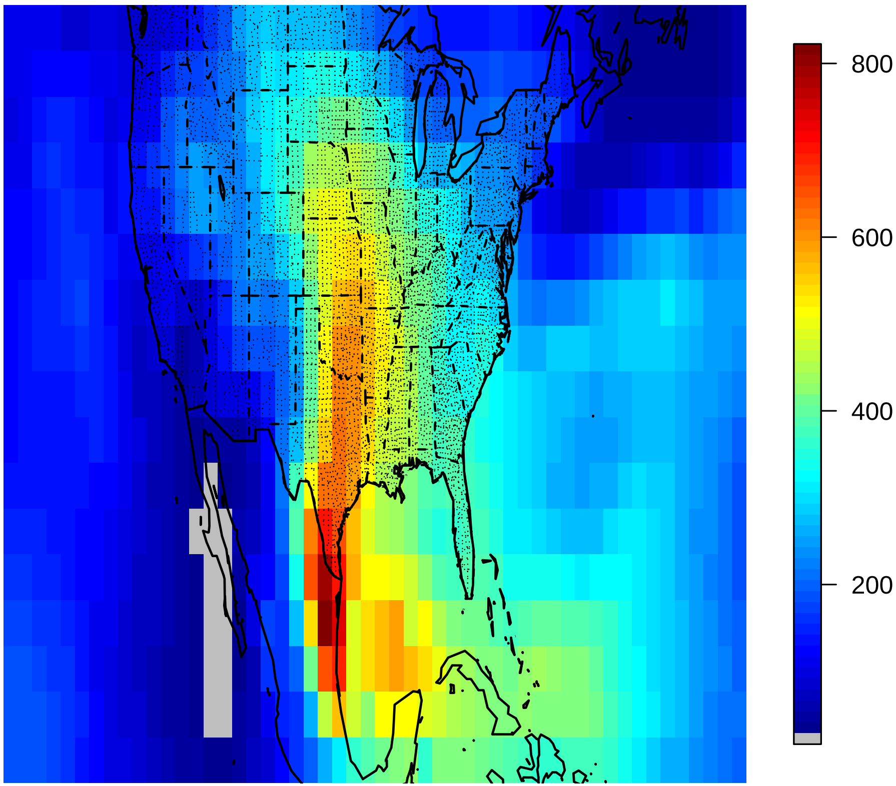
Conditional EVA

$$\Pr\left\{\frac{Y|X = x - \alpha x}{x^{\beta}} \leq z|x > u\right\} \longrightarrow G(z)$$

Joint tail behavior is characterized by α, β and G. G is not specified by theory, and there is no assumption of multivariate regular variation.

Large-scale indicators for Severe weather

Mean Predicted WmSh (m/s) conditioned on high field energy

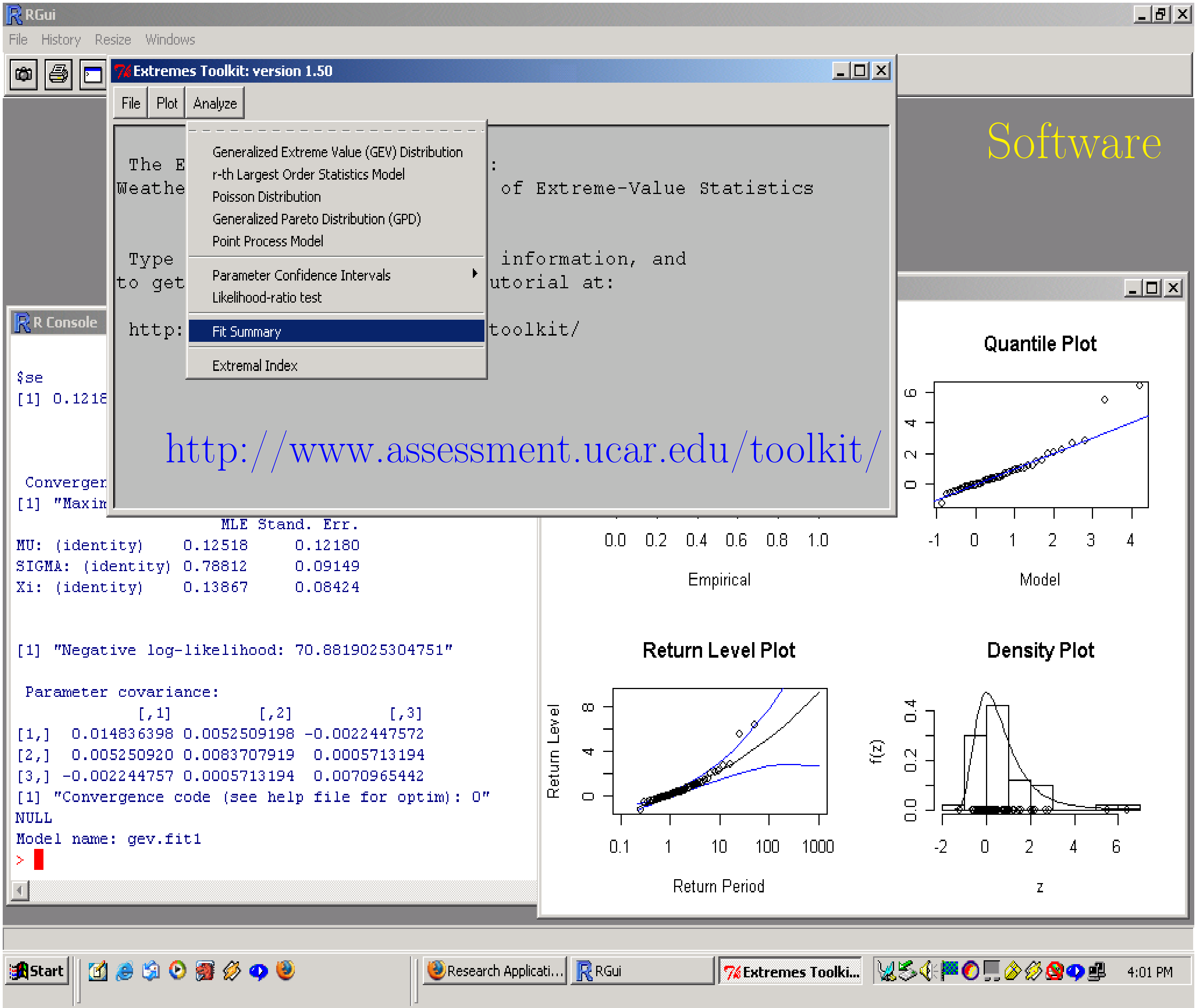


Performed using the `texmex` package in R

Summary/Conclusions/Discussion

- Defining extremes: Well-established statistical theory for events with small probabilities of occurrence, but many chances to occur.
- Weather spells are trickier to analyze: dependent on location and definition.
- Difficult to model severe weather events because of scale. Can use large-scale indicators of environments conducive to having severe weather.
- Analyzing extremes in the face of spatial dependence.
  - Multivariate EVA, Copulas, BHM, EV df's with spatial covariates. Each is valid, and can be useful, but there are important drawbacks to each.
  - Conditional EVA (Heffernan and Tawn model) shows a lot of promise. Still some drawbacks, but less important for most studies.

Software



http://www.assessment.ucar.edu/toolkit/

Discussion

- How should extreme events be defined? Deadliness? Perception-based? Statistically? Economically? Other?
- What is the relationship between changes in the mean and changes in extremes? What about variability? Higher order moments?
- If climate models project the df of atmospheric variables, then do they accurately portray the df's? Enough so that functionals of interest, such as extrema, are correctly characterized?
- If climate models only project the mean, then can anything be said about extremes?
- How can it be determined if small changes in high values of large-scale indicators lead to a shift in the df of severe weather conditional on the indicators?

Discussion

- How do we verify climate models, especially for inferring about extremes?
- Extremes are often largely dependent on local conditions (e.g., topography, surface conditions, atmospheric phenomena, etc.), as well as larger scale processes.
- Can a *metric* for climate change pertaining to extremes be developed that makes sense, and provides reasonably accurate information?
- How can uncertainty be characterized? Is there too much uncertainty to make inferences about extremes?
- How can spatial structure be taken into account for extremes?
- Many extreme events, and especially extreme impact events, result from multivariate processes. How can this be addressed?