NCAR

Introduction to Extreme Value Analysis

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Satellite Workshop on Statistical Computing for Extremes
http://sites.lsa.umich.edu/eva2015/
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National Center for Atmospheric Research

Extreme Value Analysis

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Colorado Lottery Power ball

POWER PLAY PAYOUT TABLE

MATCH	PRIZE	X2	X3	X4	X5
○○○○○●	Jackpot	POWER PLAY does not apply.			
○○○○○	$200,000	$1,000,000*			
○○○○●	$10,000	$20,000	$30,000	$40,000	$50,000
○○○○	$100	$200	$300	$400	$500
○○○●	$100	$200	$300	$400	$500
○○○	$7	$14	$21	$28	$35
○○●	$7	$14	$21	$28	$35
○●	$4	$8	$12	$16	$20
●	$3	$6	$9	$12	$15

Probability of winning the jackpot ≈ 5.7×10^{-9}

https://www.coloradolottery.com/games/powerball/

Suppose we play one ticket every day for ten years, what is the probability of winning the lottery at least one time?

Law of small numbers gives the Poisson distribution as a good approximation to such a probability. Here, the rate parameter is (5.7×10^{-9})(10 years)(365.25 days/year)

Extreme Value Analysis

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Probability of winning the jackpot ≈ 5.7×10^{-9}

https://www.coloradolottery.com/games/powerball/

Suppose we play one ticket every day for ten years, what is the probability of winning the lottery at least one time?

So, the above probability is approximated by
$1 - \exp((5.7 \times 10^{-9})(10)(365.25)) \approx 2.08 \times 10^{-5}$

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Colorado Lottery Power ball

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Probability of winning the jackpot ≈ 5.7×10^{-9}

https://www.coloradolottery.com/games/powerball/

How long can I expect to wait until I win?

The waiting time distribution for this scenario is governed by the exponential distribution with mean equivalent to the reciprocal of the associated Poisson intensity parameter.

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Colorado Lottery Power ball

POWER PLAY PAYOUT TABLE

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●	$3	$6	$9	$12	$15

Probability of winning the jackpot ≈ 5.7×10^{-9}

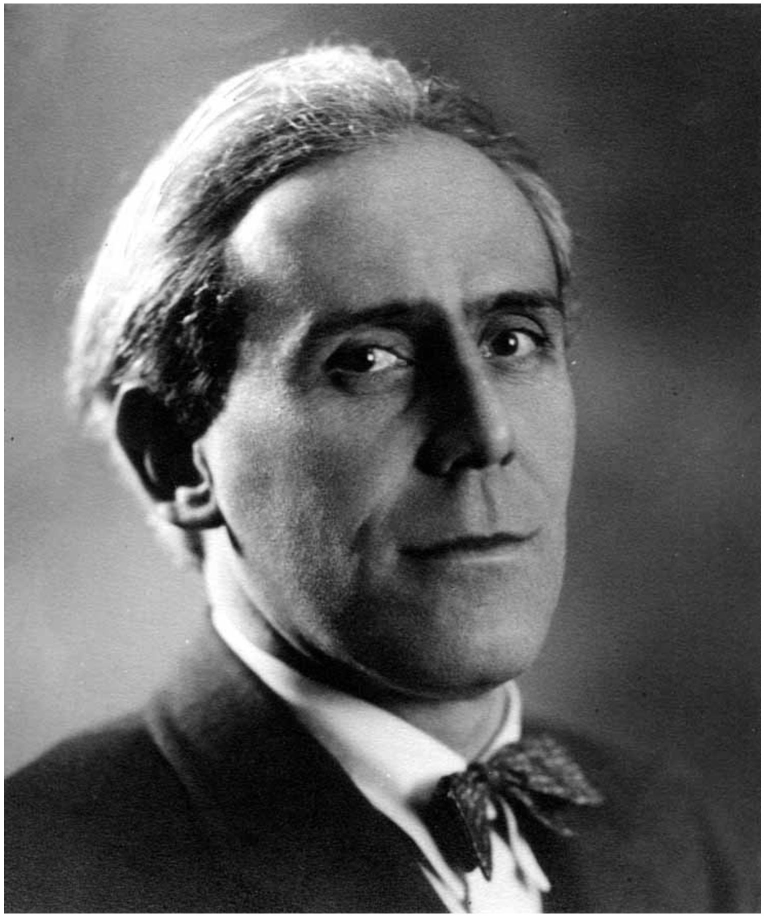
https://www.coloradolottery.com/games/powerball/

How long can one expect to wait until they win?

In this case, it is expected to wait on average over 48 000 years.

Extreme Value Analysis

"Il est impossible que l'improbable n'arrive jamais"
--Emil Gumbel



Extreme Value Analysis

Events do not need to be as rare as winning the lottery to follow the Poisson distribution.

Require that the (binomial) probability of success tends to zero at a fast enough rate that the expected number of events is constant.

Extreme Value Analysis

Let $X_1, \ldots, X_m$ be a series of iid random variables with distribution F. Let N denote the number of events where X_t exceeds a high threshold u_n over n days.

N has an approximate Poisson distribution with intensity parameter $n\lambda$ and

$$\Pr\{N = 0\} = \exp(-n\lambda) \quad \text{or} \quad \Pr\{N > 0\} = 1 - \exp(-n\lambda)$$

In general, want $n \Pr\{N > u_n\} \longrightarrow \lambda$ as $n \longrightarrow \infty$

where u_n is increasing as n increases.

Extreme Value Analysis

Suppose we want to characterize the distribution for extreme values rather than frequencies of rare events?

Let $X_1, \ldots, X_n$ denote a series of iid random variables, and let $M_n = \max\{X_1, \ldots, X_n\}$.

Now consider the event that a realization of this series exceeds a high threshold, u_n, and let N_n denote the number of such events in n realizations.

Then, we have that $\Pr\{N_n = 0\} = \Pr\{M_n \leq u_n\}$.

Extreme Value Analysis

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Suppose we want to characterize the distribution for extreme values rather than frequencies of rare events?

Another route:

Let F be the cdf of $X_1, \ldots, X_n$. Then $\Pr\{M_n \leq z\} = F^n(z)$.

But $F^n(z)$ tends to zero as n tends to infinity. Also, F is generally unknown, and small discrepancies in F can lead to large discrepancies in F^n.

Extreme Value Analysis

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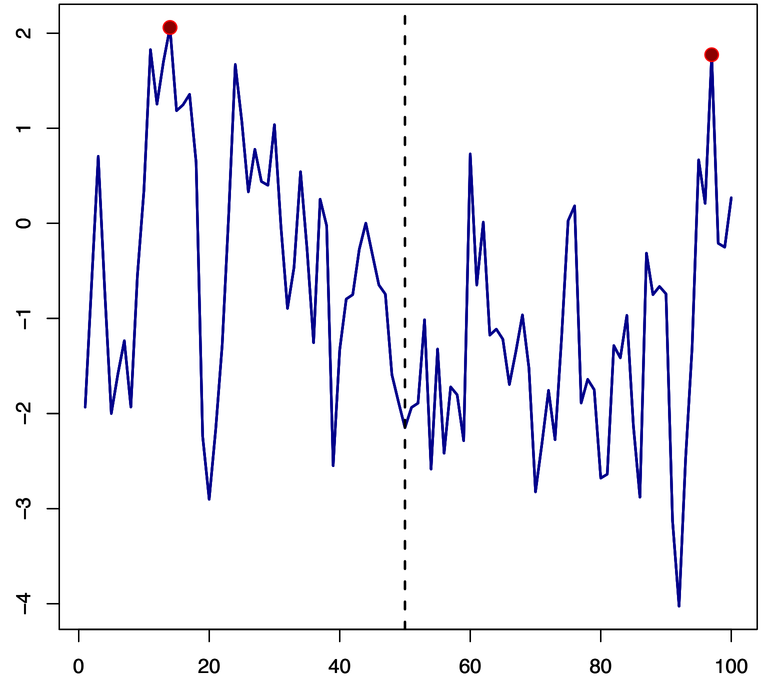
Max Stability

$\max\{x_1, \ldots, x_{100}\} =$

$\max\{\max\{x_1, \ldots, x_{50}\}, \max\{x_{51}, \ldots, x_{100}\}\}$



More precisely, a distribution F is said to be *max stable* if, for every n = 2, 3, …, there exist sequences of constants $a_n > 0$ and b_n such that

$$F^n(a_n z + b_n) = F(z)$$

Extreme Value Analysis

Suppose we want to characterize the distribution for extreme values rather than frequencies of rare events?

Another route:

Let F be the cdf of $X_1, \ldots, X_n$. Then $\Pr\{M_n \leq z\} = F^n(z)$.

But $F^n(z)$ tends to zero as n tends to infinity. Also, F is generally unknown, and small discrepancies in F can lead to large discrepancies in F^n.

Trick is to find sequences of constants $a_n > 0$ and b_n, such that $F^n((z - b_n) / a_n)$ tends to a non-degenerate distribution as n tends to ∞.

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Generalized Extreme Value (GEV) distribution function

$$\Pr\{M_n \le z\} = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$$

$\mu \in \mathbb{R}$ (location parameter)

$\sigma > 0$ (scale parameter)

$\xi \in \mathbb{R}$ (shape parameter)

Look familiar?

Defined where the part inside the [ ] is positive.

Extreme Value Analysis

Generalized Extreme Value (GEV) distribution function

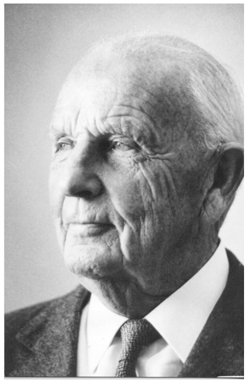
Three Types

Type I: Gumbel (light tail, shape = 0)
domain of attraction for many common distributions

Type II: Fréchet (heavytail, shape > 0)
precipitation, stream flow, economic impacts
Infinite mean if shape parameter ≥ 1
Infinite variance if shape parameter ≥ 0.5

M. R. Fréchet

Type III: (reverse) Weibull (bounded upper tail, shape < 0)
temperature, wind speed, sea level

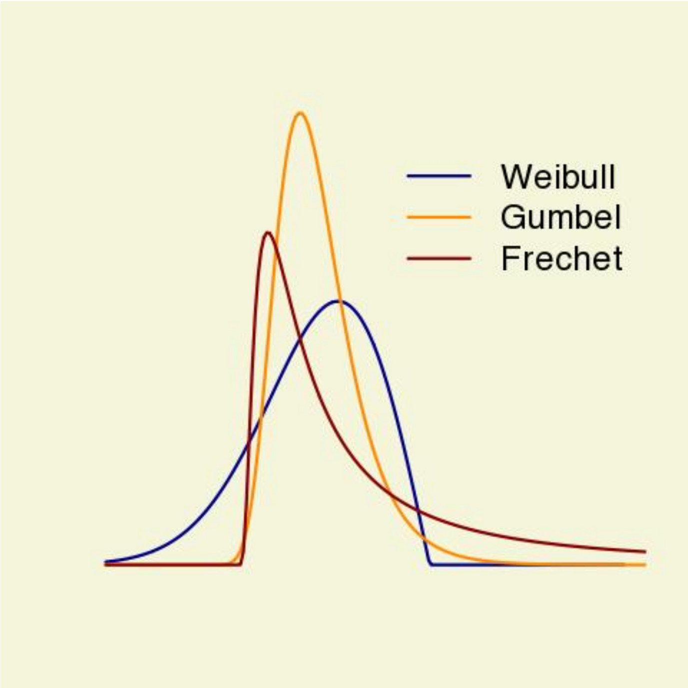


E. H. Waloddi Weibull

Extreme Value Analysis

Generalized Extreme Value (GEV) distribution function

Three Types



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Another characterization is to look at excesses over a threshold.

Analogous to block maxima, but now the generalized Pareto distribution holds.

Three types are now: Exponential (zero shape), Pareto (positive shape) and Beta (negative shape).

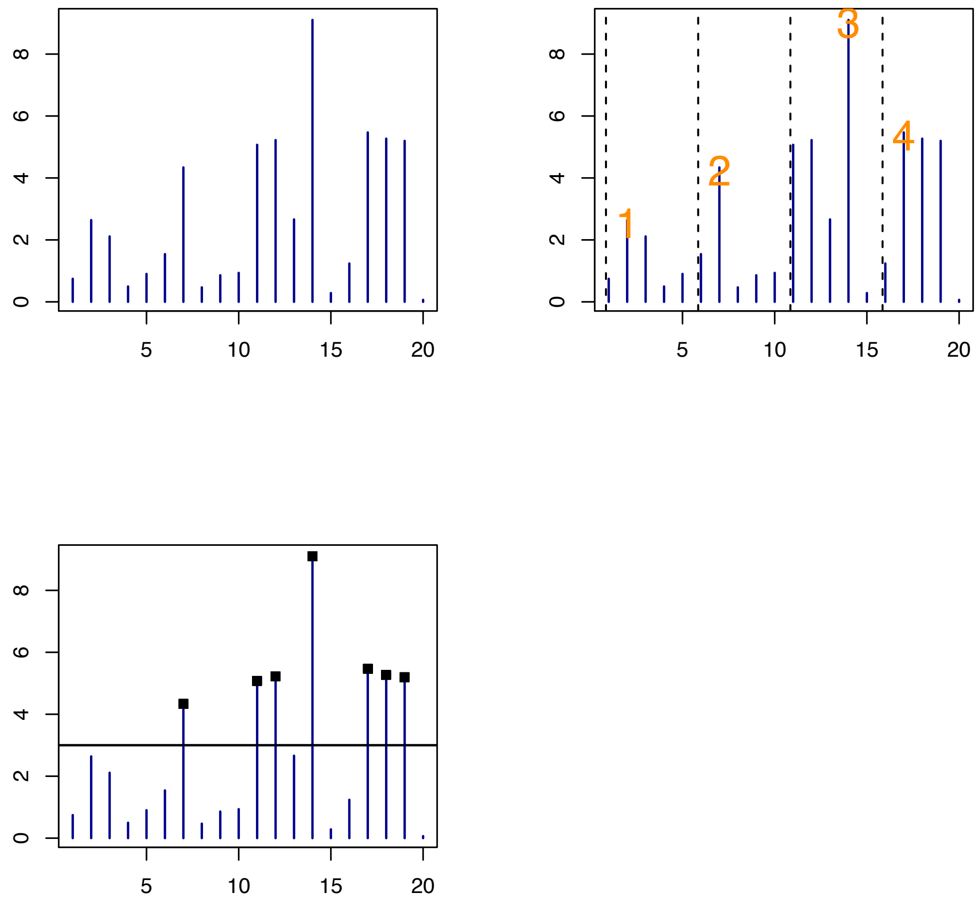
$$\Pr\left\{X-u \middle| X>u\right\} = \left[1+\xi\frac{x-u}{\sigma}\right]_{+}^{-1/\xi}$$

Extreme Value Analysis

Yet another characterization is to simultaneously look at the frequency and value of the rare events.

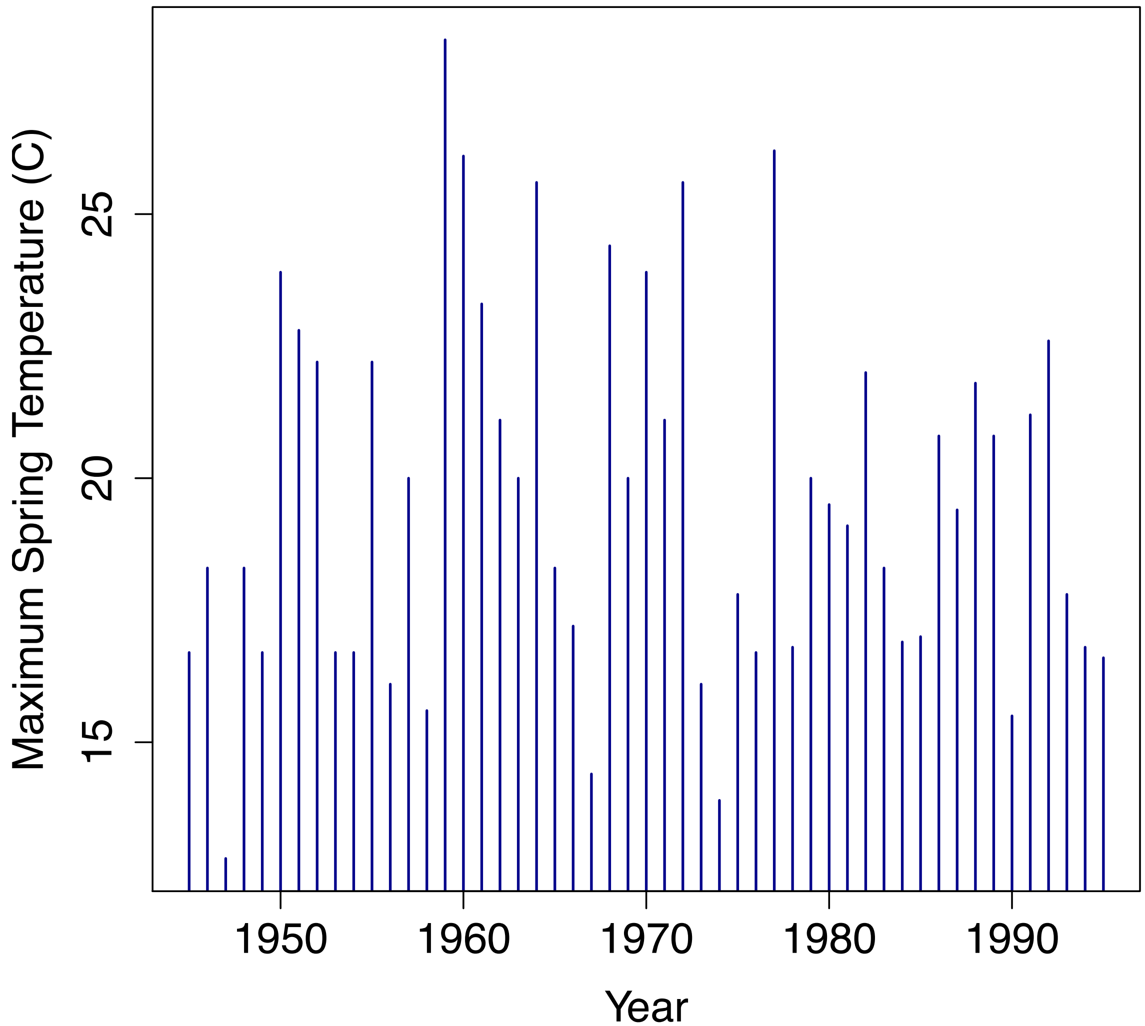
Point Process (PP) approach.

Extreme Value Analysis



Extreme Value Analysis

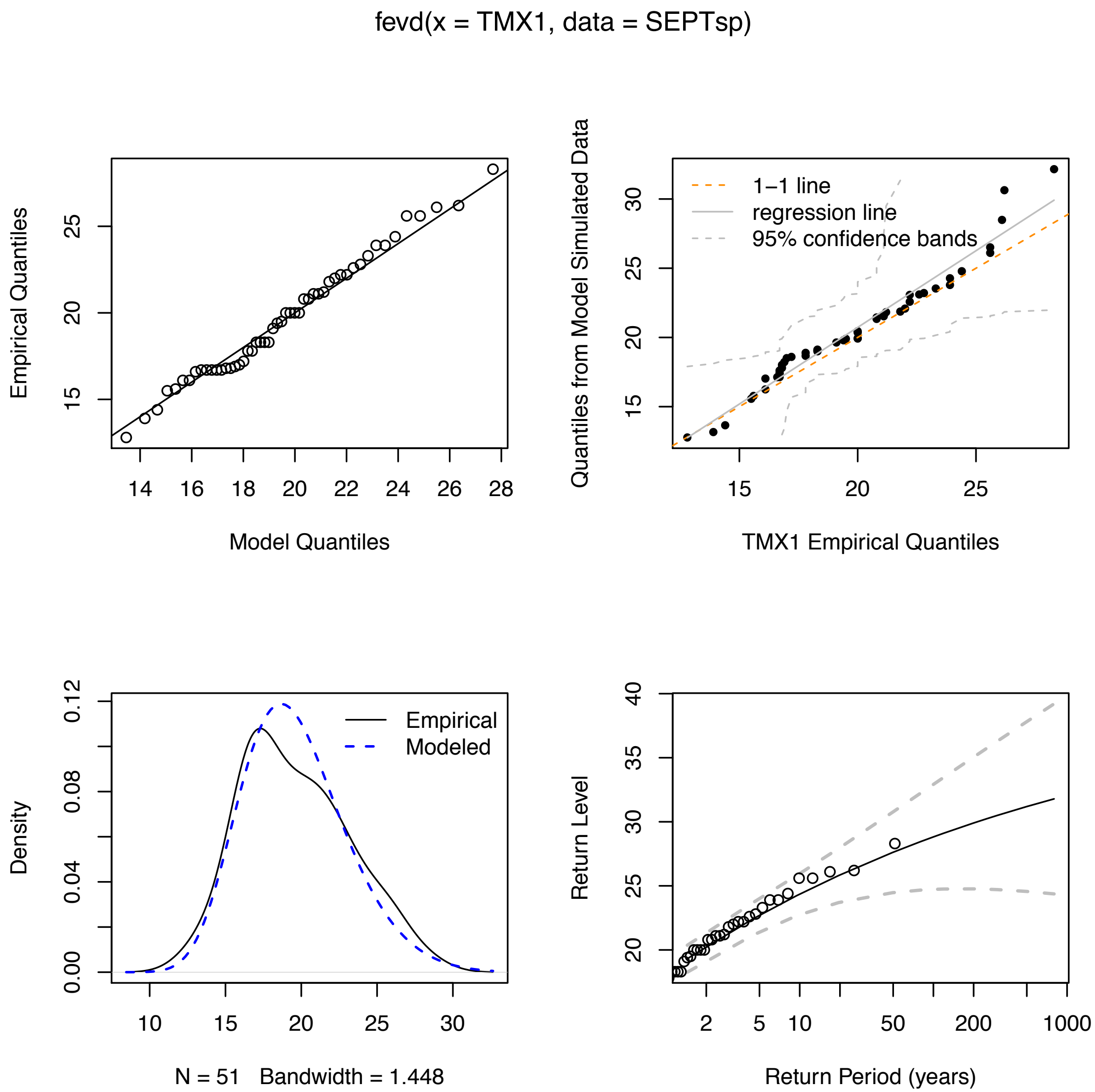
Data Example



Sept, Iles, Québec

Extreme Value Analysis

Data Example



Sept, Iles, Québec

Extreme Value Analysis

Data
Example

	95% lower CI	Estimate	95% upper CI
μ	17.22	18.20	19.18
σ	2.42	3.13	3.84
ξ	-0.37	-0.14	0.09
100-year return level	24.72 °C	28.81 °C	32.90 °C

Sept, Iles, Québec

Extreme Value Analysis

Assume stationarity (i.e. unchanging climate)

Return period / Return Level

Seek x_p such that $G(x_p) = 1 - p$, where $1 / p$ is the return period. That is,

$x_p = G^{-1}(1 - p; \mu, \sigma, \xi)$, $0 < p < 1$

Easily found for the GEV cdf.

Example, $p = 0.01$ corresponds to 100-year return period (assuming annual blocks).

Extreme Value Analysis

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What if the data are:

- Not stationary?
- Stationary, but have a seasonal, diurnal, etc. cycle?

Usual solution is to model one or more of the parameters with a covariate.

Can be accomplished easily under the existing framework, though it can make interpretation of return levels more difficult.

Extreme Value Analysis

Suppose, additionally, a model is imposed on the parameters.

For example, given a spatio-temporal process, $Z(\boldsymbol{x}, t)$, what can be said about

$$\Pr\{ Z(\boldsymbol{x}, t) > z\}$$

when z is large?

Extreme Value Analysis

More general situation:

Suppose, additionally, a model is imposed on the parameters.

For example, given a spatio-temporal process, $Z(\boldsymbol{x}, t)$, what can be said about

$$\Pr\{ Z(\boldsymbol{x}, t) > z\}$$

when z is large?

Extreme Value Analysis

Example

Observation model: $Z(\boldsymbol{x}, t)$ a space-time process at location $\boldsymbol{x}$ and time t.

$$[Z(\boldsymbol{x}, t) \mid \sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u, Z(\boldsymbol{x}, t) > u]$$

Spatial process model:

$$[\sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u \mid \boldsymbol{\theta}]$$

Prior for hyper parameters:

$$[\boldsymbol{\theta}]$$

Extreme Value Analysis

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Example

Assume extreme observations to be *conditionally independent* so that the joint pdf for the data and parameters is

$$\prod_{i,t} [Z(\boldsymbol{x}_i, t) \mid \sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u, Z(\boldsymbol{x}, t) > u][\sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u \mid \boldsymbol{\theta}][\boldsymbol{\theta}]$$

Note: such conditional independence is often not met; at least in geophysical applications

Dependence is accounted for in terms of the model parameters, but not between $Z(\boldsymbol{x}_i, t)$ and $Z(\boldsymbol{x}_j, t + \tau)$

Extreme Value Analysis

Example

Assume extreme observations to be *conditionally independent* so that the joint pdf for the data and parameters is

$$\prod_{i,t} [Z(\boldsymbol{x}_i, t) \mid \sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u, Z(\boldsymbol{x}, t) > u][\sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u \mid \boldsymbol{\theta}][\boldsymbol{\theta}]$$

Reasonable short-cut assumptions:

- $\xi(\boldsymbol{x}) = \xi$ (or just don't impose any model on ξ)
- $\sigma(\boldsymbol{x})$ is a Gaussian process with isotropic Matern covariance function
- Fix Matern smoothness parameter at 2, and let the range be very large; leaving only λ (ratio of variances of nugget and sill).

Extreme Value Analysis

Example

Assume extreme observations to be *conditionally independent* so that the joint pdf for the data and parameters is

$$\prod_{i,t} [Z(\boldsymbol{x}_i, t) \mid \sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u, Z(\boldsymbol{x}, t) > u][\sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u \mid \boldsymbol{\theta}][\boldsymbol{\theta}]$$

- λ is the only hyper-parameter (assume an uninformative prior).
- $\sigma(\boldsymbol{x}) = P(\boldsymbol{x}) + e(\boldsymbol{x}) + \eta(\boldsymbol{x})$, where P is a linear function of space, e a smooth spatial process, and η white noise.
- $\lambda \rightarrow \infty$; the posterior surface tends toward the linear function.
- $\lambda \rightarrow 0$; the posterior surface fits the data more closely.

Extreme Value Analysis

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Example

Assume extreme observations to be *conditionally independent* so that the joint pdf for the data and parameters is

$$\prod_{i,t} [Z(\boldsymbol{x}_i, t) \mid \sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u, Z(\boldsymbol{x}, t) > u][\sigma(\boldsymbol{x}), \xi(\boldsymbol{x}), u \mid \boldsymbol{\theta}][\boldsymbol{\theta}]$$

log of joint distribution:

covariance for the prior on σ at the observations

$$\sum_{i=1}^{n} \ell_{\text{GPD}}\left(Z\left(\mathbf{x}_i, t\right), \sigma\left(\mathbf{x}_i\right), \xi\right) - \lambda\left(\sigma - \mathbf{X}\beta\right)^T K^{-1}\left(\sigma - \mathbf{X}\beta\right)/2 - \log\left(\left|\lambda K\right|\right) + C$$

It is now a penalized likelihood!

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Similarly for multivariate extremes:

$\mathbf{M}_n = (\max\{X_1, \ldots, X_n\}, \max\{Y_1, \ldots, Y_n\}) / n$

If a non-degenerate limiting distribution exists, then it must have the form:

$$G(x,y) = \exp\left[-V(x,y)\right], \text{ where } x>0, y>0 \text{ and}$$

$$V(x,y) = 2\int_0^1 \max\left(\frac{w}{x}, \frac{1-w}{y}\right) dH(w), \text{ with}$$

$$\int_0^1 w\, dH(w) = \frac{1}{2}$$

Note: This is for X and Y suitably transformed to a unit scale (usually unit Frechet)

Extreme Value Analysis

Idea of multivariate extremes is related to copula modeling where the dependence is measured on the transformed variates through a “copula” dependence model.

That is, instead of measuring the dependence between X and Y, measure the dependence between

$F_X(X)$ and $F_Y(Y)$

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Conditional Model (Heffernan and Tawn, 2004, JRSS B, 66, 497 – 546; cf. also Heffernan and Resnick, 2007, Annals of Applied Probability, 17, 527 – 571).

Opening Assumption: For X and Y suitably transformed to a common scale:

$$\Pr\left\{Y - u > y, \frac{X - a(Y)}{b(Y)} \leq z \middle| Y > u\right\} \to \exp(-y)G(z) \text{ as } u \to \infty$$

Extreme Value Analysis

$$\Pr\left\{Y-u>y,\frac{X-a(Y)}{b(Y)}\le z \middle| Y>u\right\}\to\exp(-y)G(z)\text{ as }u\to\infty$$

For a wide class of copula models, Heffernan and Tawn showed that:

$$a(Y)=\alpha Y \qquad \alpha\in[0,1]$$

$$b(Y)=Y^{\beta} \qquad \beta\in(-\infty,1)$$

If using the Laplace transform, then $\alpha\in[-1,1]$

(e.g. Keef et al. 2013, J. Multivariate Analysis, 115, 396 – 404)

Extreme Value Analysis

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$$\Pr\left\{Y-u>y, \frac{X-a(Y)}{b(Y)} \le z \middle| Y>u\right\} \to \exp(-y)G(z) \text{ as } u \to \infty$$

- Y is conditioned to be extreme in this model, but X may or may not be extreme.
- Implied independence from the initial assumption. In particular, cannot usefully turn the conditioning around to examine the extremes of Y given X.
- No simple closed-form expression for G, in general.
- Useful expression: $X_{|Y>u} = \alpha Y + Y^{\beta} Z_{|Y>u}$

Thank You
End of Part I

Questions?

Next: R software packages

Then: Performing EVA using extRemes

Part II: R software packages

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- Web page with summary of EVA software:
  - http://www.ral.ucar.edu/staff/ericg/softextreme.php
- Review papers
  - G. et al. (2013, Extremes, 16, 103 – 119)
  - Stephenson and G. (2005, Extremes, 8, 87 – 109)

Primary R packages for EVA

- Univariate EVA
  - evir
  - extRemes, in2extRemes
  - fExtremes
  - ismev
  - lmom, lmomRFA, lmomco
  - texmex
  - VGAM
- Multivariate EVA
  - copula
  - evd, evdbayes
  - evir
  - lmom, lmomco
  - SpatialExtremes, RandomFields
  - texmex

	Block Maxima	POT	Estimation methods	Parameter Covariates	Multi-variate?
copula			MLE, pseudo MLE, MOM		Yes
evd	Yes	Yes	MLE	some	bivariate
evdbayes	Yes	Yes	Bayesian	limited	
extRemes	Yes	Yes	MLE, LM, GMLE, Bayesian	Yes	No
evir	Yes	Yes	MLE		limited
fExtremes	Yes	Yes	MLE, PWM		
ismev	Yes	Yes	MLE	Yes	
lmom	Yes	Yes	LM		
lmomRFA, lmomco	Yes	Yes	LM		limited, Yes
SpatialExtremes	Yes	Yes	MLE, MCLE, Bayesian	Yes	Yes
texmex	Yes	Yes	MLE, PMLE, Bayesian	Yes	Yes
VGAM	Yes	Yes	MLE, BFA	Yes	No

Update of Table 1 in G. et al (2013)

Other Relevant R packages

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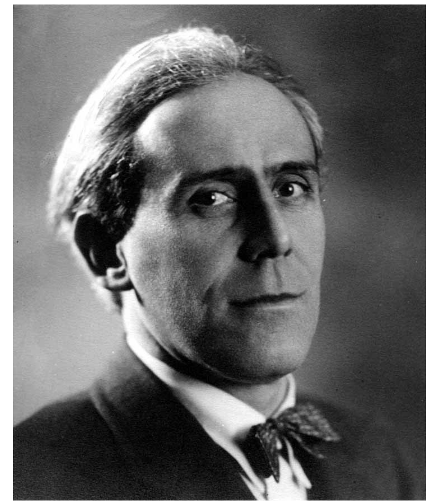
- ABCExtremes
- acer
- actuar
- bgeva
- BMAevt
- BSquare
- cogarch, fGarch, gogarch, rmgarch
- CreditMetrics
- eventstudies
- evmix
- extremevalues
- extWeibQuant
- MCMC4Extremes
- QRM
- quantreg
- spatial.gev.bma
- TestEVC1d.r

Part III: Examples of Analyzing Extremes (using extRemes)

a) block maxima

Midwest flood 1993 (NCAR Digital Image Library, DI00578)

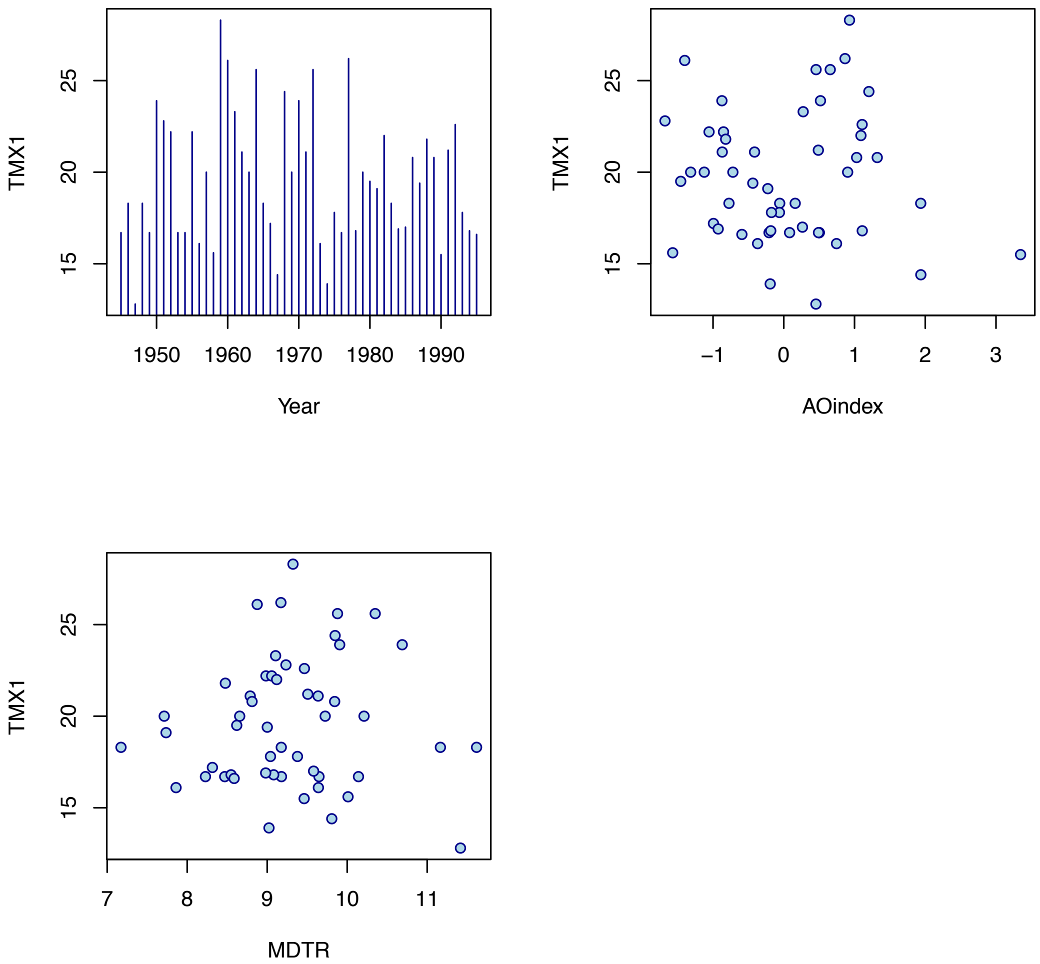


"Il est impossible que l'improbable n'arrive jamais"
--Emil Gumbel

Introduction to EVA: Examples

```
data("SEPTsp")

?SEPTsp

par(mfrow = c(2, 2))

plot(TMX1~ Year, data = SEPTsp,
    type = "h", col = "darkblue")

plot(TMX1~ AOindex, data = SEPTsp,
    pch = 21, col = "darkblue",
    bg = "lightblue")

plot(TMX1~ MDTR, data = SEPTsp,
    pch = 21, col = "darkblue",
    bg = "lightblue")
```

Introduction to EVA: Examples



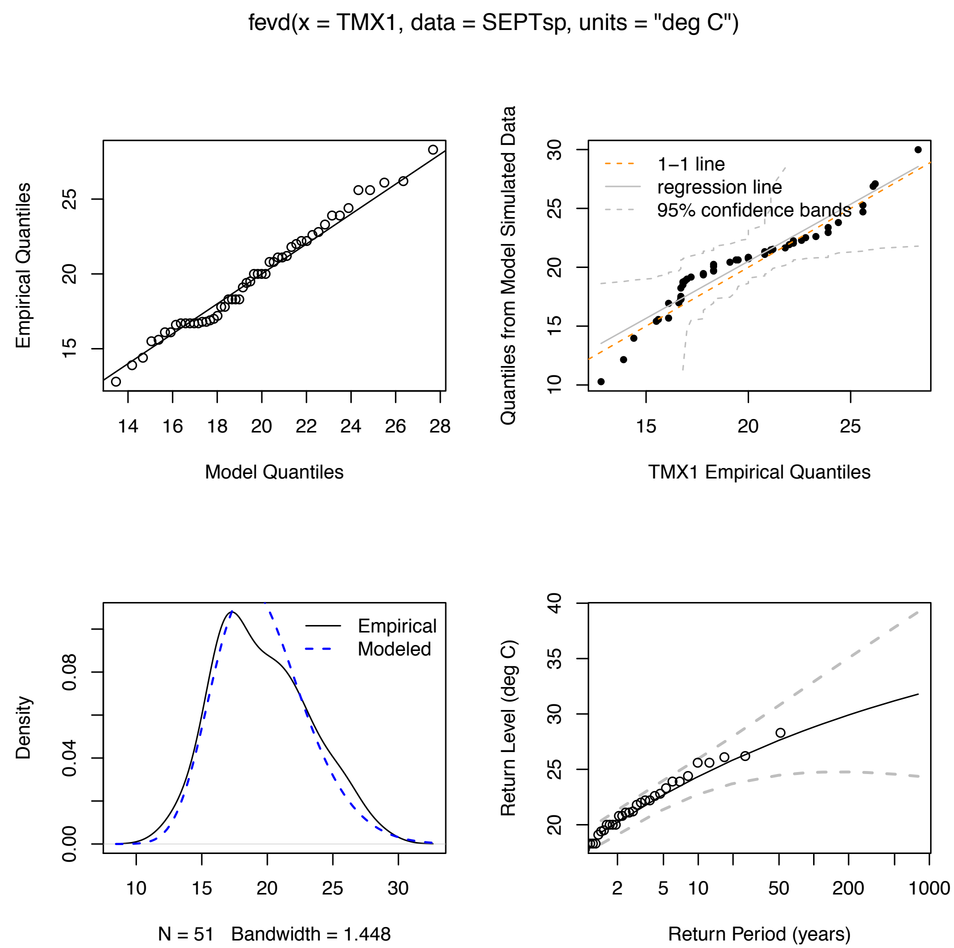
Introduction to EVA: Examples

Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec

```
fit0 <- fevd(TMX1, data = SEPTsp,
    units = "deg C")

fit0

plot(fit0)

ci(fit0, type = "parameter")

ci(fit0)
```

Introduction to EVA: Examples

Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec



Introduction to EVA: Examples

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Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec

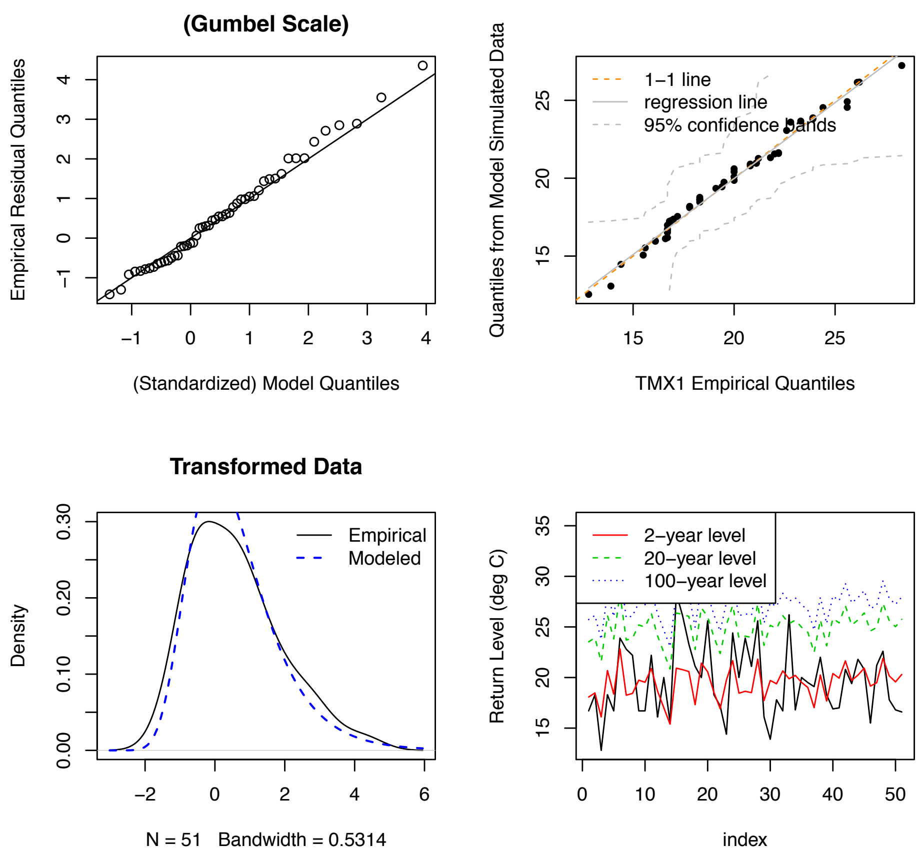
```
fit1 <- fevd(TMX1, data = SEPTsp,
    location.fun = ~AOindex,
    units = "deg C")

fit1

plot(fit1)

lr.test(fit0, fit1)
```

fits model with
$\mu(\text{AO index}) = \mu_0 + \mu_1 \times (\text{AO index})$

Introduction to EVA: Examples

Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec

fevd(x = TMX1, data = SEPTsp, location.fun = ~STDTMAX, units = "deg C")



Introduction to EVA: Examples

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Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec

```
fit2 <- fevd(TMX1, data = SEPTsp,
    location.fun = ~AOindex,
    scale.fun = ~STDTMAX,
    use.phi = TRUE,
    units = "deg C")

fit2

plot(fit2)

lr.test(fit0, fit2)
```

fits model with
$\mu(\text{AO index}) = \mu_0 + \mu_1 \times (\text{AO index})$
$\ln(\sigma(\text{AO index})) = \varphi_0 + \varphi_1 \times (\text{AO index})$

Introduction to EVA: Examples

Fit a GEV distribution to maximum spring temperature in Sept-Iles, Québec

To do the same fit using **ismev**

```
library( "ismev" )

fit0WITHismev <- gev.fit( SEPTsp$TMX1 )
gev.diag( fit0WITHismev )

fit1WITHismev <- gev.fit( SEPTsp$TMX1,
    ydat = SEPTsp, mul =  6 )

fit2WITHismev <- gev.fit( SEPTsp$TMX1,
    ydat = SEPTsp, mul =  6,
    sigl = 6, siglink = exp )
```

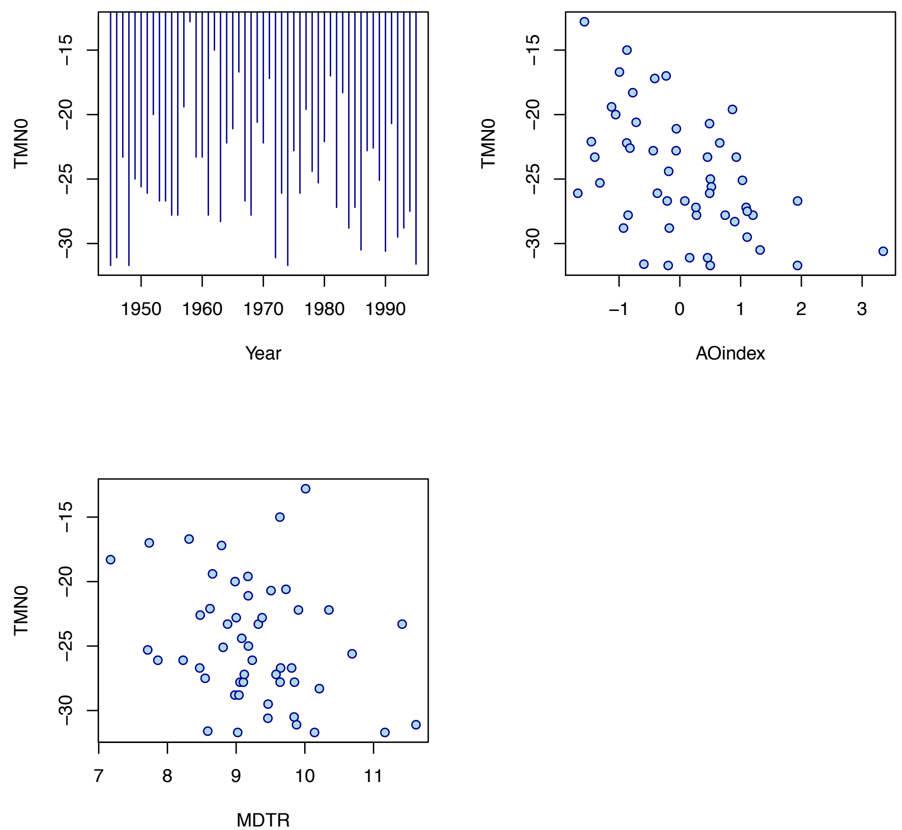
Introduction to EVA: Examples

__Minimum__ spring temperature (deg. C) in Sept-Iles, Québec

```
par(mfrow = c(2, 2))

plot(TMN0~ Year, data = SEPTsp,
    type = "h", col = "darkblue")

plot(TMN0~ AOindex, data = SEPTsp,
    pch = 21, col = "darkblue",
    bg = "lightblue")

plot(TMN0~ MDTR, data = SEPTsp,
    pch = 21, col = "darkblue",
    bg = "lightblue")
```

Introduction to EVA: Examples

Minimum spring temperature (deg. C) in Sept-Iles, Québec



Introduction to EVA: Examples

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Minimum spring temperature (deg. C) in Sept-Iles, Québec

```
fit0 <- fevd(-TMN0 ~ 1, data = SEPTsp,
    units = "neg. deg. C")

fit0

plot(fit0)
```

The rest of fitting the GEV to negative minimum temperature is left as an exercise

Introduction to EVA: Examples

b) Frequency of extremes

photo from Wikipedia: http://en.wikipedia.org/wiki/Coligny_calendar

Introduction to EVA: Examples

b) Frequency of extremes

Number of days that maximum daily temperature (deg. F) in Fort Collins, Colorado exceeds 95 degrees F.

```
data("FCwx")

?FCwx

tempGT95 <- c(aggregate(FCwx$MxT,
    by = list(FCwx$Year),
    function(x) sum(x > 95, na.rm = TRUE))$x)

yr <- unique(FCwx$Year)
```

Introduction to EVA: Examples

b) Frequency of extremes

Number of days that maximum daily temperature (deg. F) in Fort Collins, Colorado exceeds 95 degrees F.

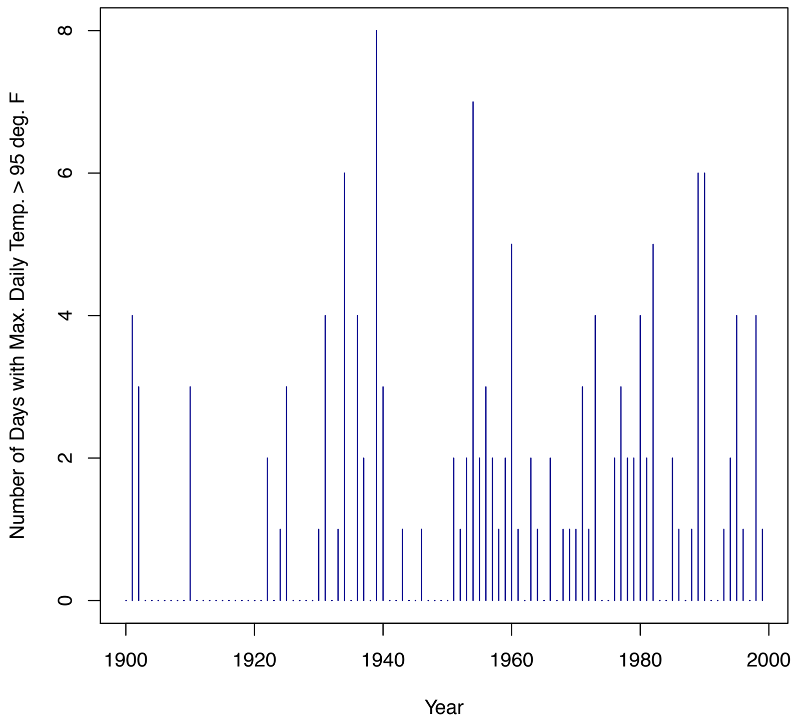
```
plot(yr, tempGT95, type = "h",
    col = "darkblue", xlab = "Year",
    ylab =
    ”No. of Days with Max. Daily Temp. > 95 deg. F")


fpois(tempGT95)
```

Introduction to EVA: Examples

b) Frequency of extremes

Number of days that maximum daily temperature (deg. F) in Fort Collins, Colorado exceeds 95 degrees F.



Introduction to EVA: Examples

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b) Frequency of extremes

Number of days that maximum daily temperature (deg. F) in Fort Collins, Colorado exceeds 95 degrees F.

```
fit <- glm(tempGT95~yr, family = poisson())

summary(fit)
```

Introduction to EVA: Examples

c) Threshold excesses

V.F.D. Pareto

Introduction to EVA: Examples

c) Threshold excesses

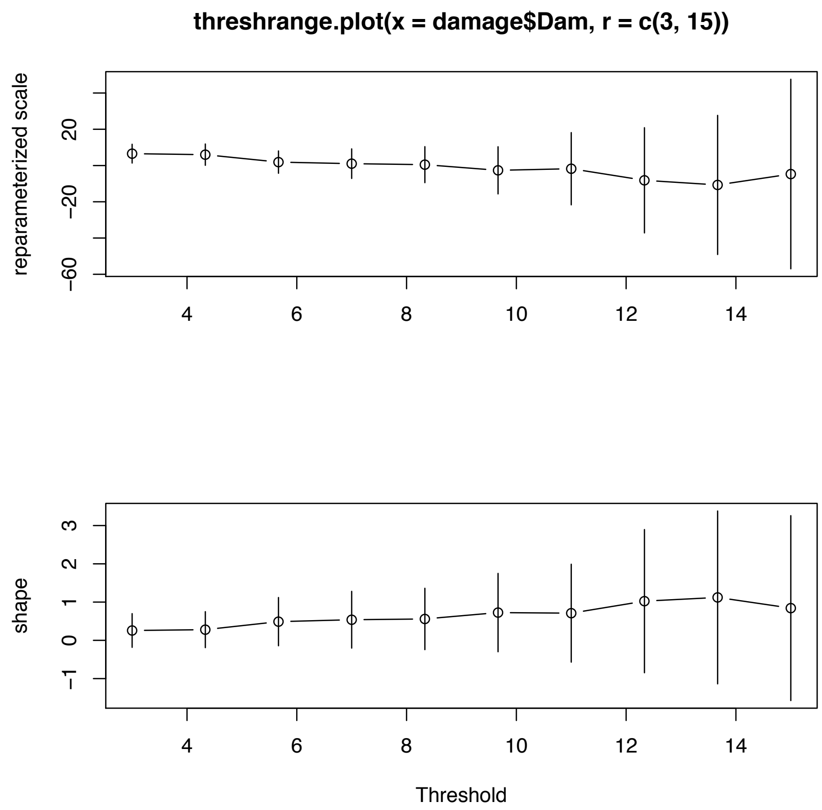
Hurricane Damage (billion USD, 1925 – 1995)

```
data( "damage" )

sum( damage$Dam > 0 )

144 / 71

threshrange.plot( damage$Dam, r = c(3, 15) )

atdf( damage$Dam, u  = 0.8 )

extremalindex( damage$Dam, threshold = 6 )
```

Introduction to EVA: Examples

c) Threshold excesses

Hurricane Damage (billion USD, 1925 – 1995)

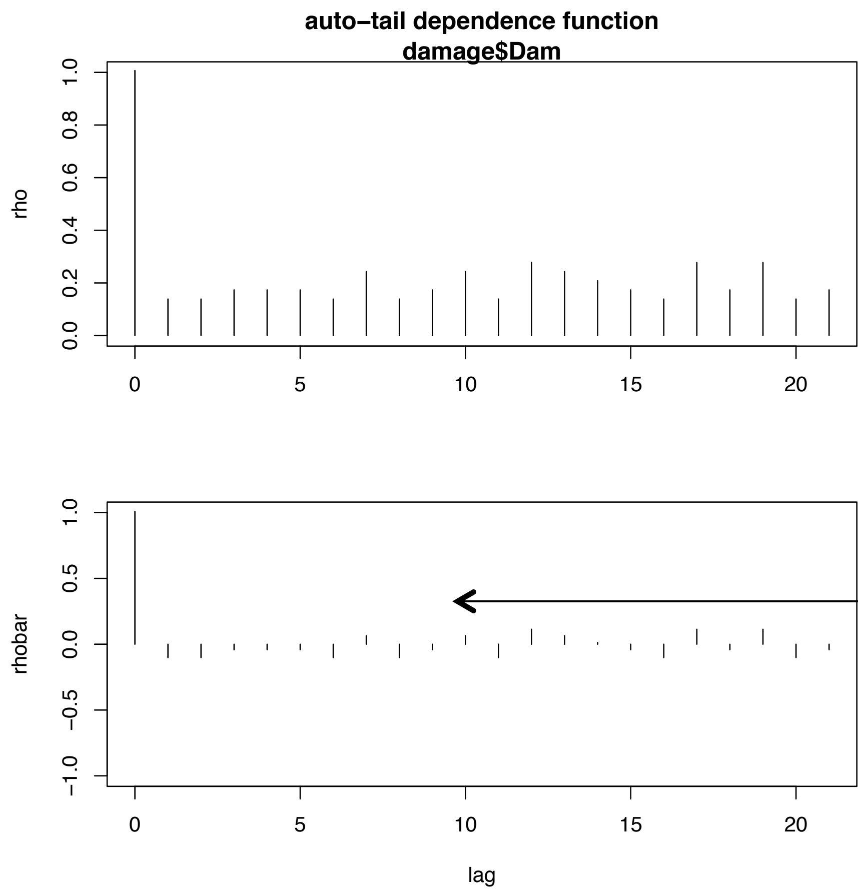


Introduction to EVA: Examples

c) Threshold excesses

Hurricane Damage (billion USD, 1925 – 1995)

0.2 = 1 – 0.8



appears to be tail independent

Perfect dependence if equal to unity, tail independence with value showing strength of dependence.

Introduction to EVA: Examples

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c) Threshold excesses

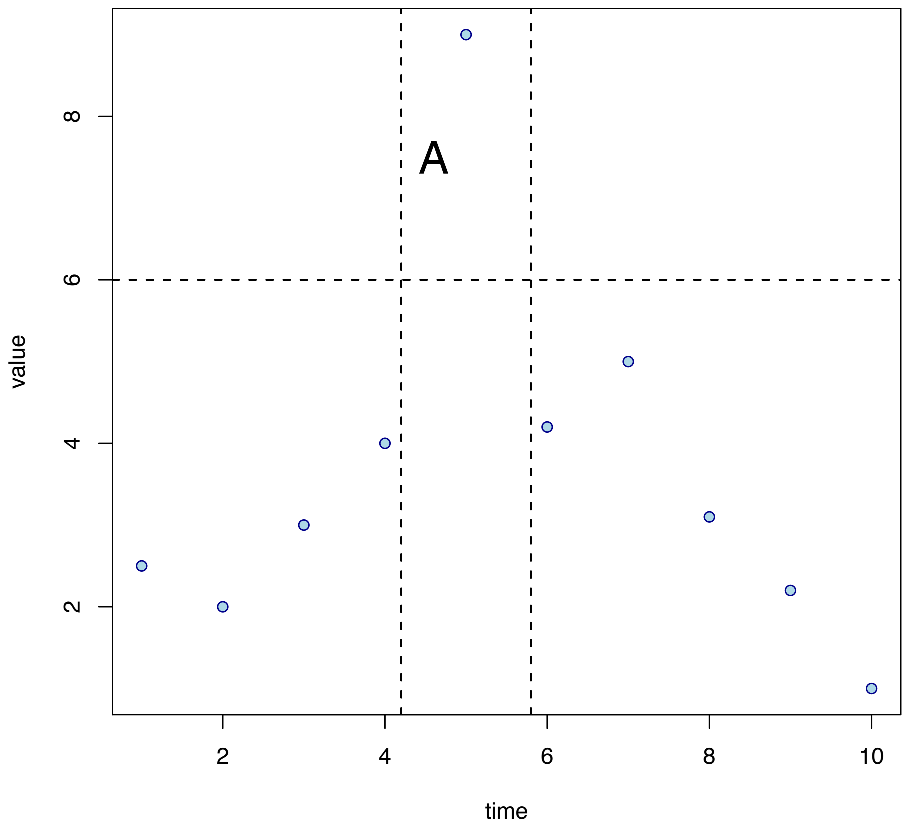
Hurricane Damage (billion USD, 1925 – 1995)

```
fit0 <- fevd( Dam, threshold = 6, data = damage,
    type = "GP", units = "billion USD”,
    time.units = "2.03/year" )

fit0

plot( fit0 )
```

Important to get the return levels correct!

Introduction to EVA: Examples

d) Point process



Siméon Denis Poisson

Introduction to EVA: Examples

d) Point process

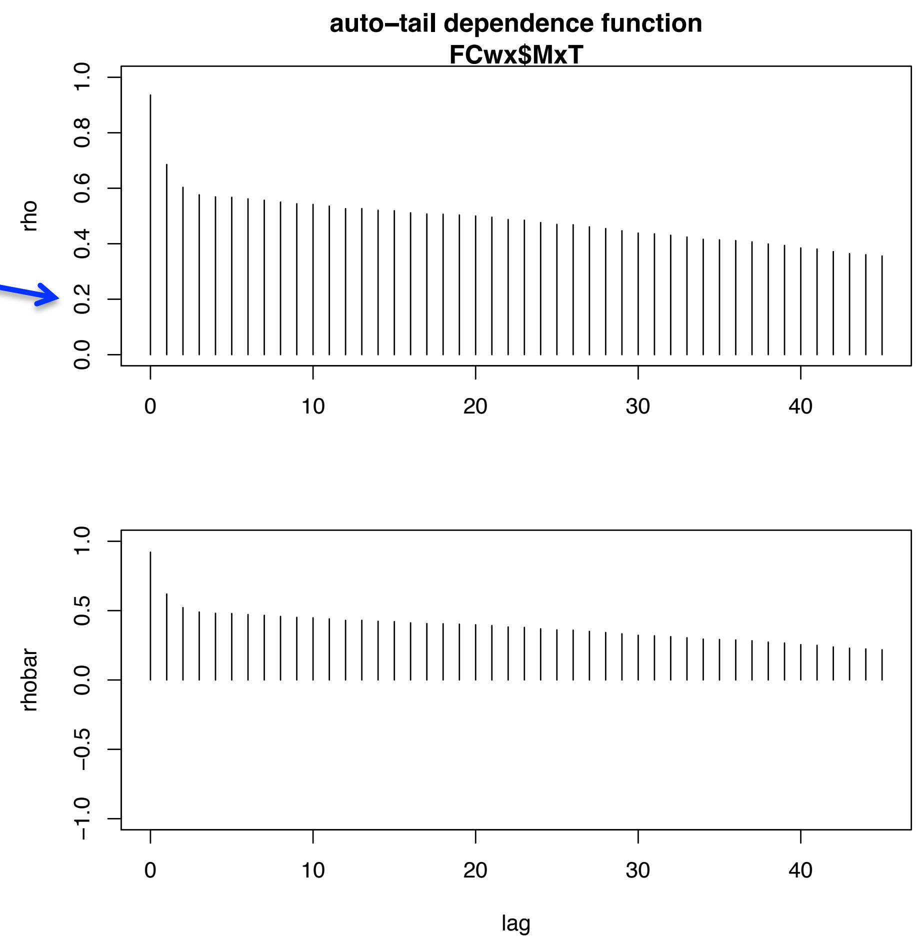
```
plot(MxT~ Year, data = FCwx,
    pch = 21, col = "darkblue",
    bg = "lightblue")

plot(MxT~ Mn, data = FCwx,
    pch = 21,
    col = "darkblue",
    bg = "lightblue")

atdf(FCwx$MxT, 0.8)
```

No obvious annual trend, but a clear seasonal cycle. Appears to have dependence issues in the threshold excesses.

Introduction to EVA: Examples

d) Point process

0.2 = 1 – 0.8



Exhibits tail dependence.

Perfect dependence if equal to unity, tail independence with value showing strength of dependence.

Introduction to EVA: Examples

d) Point process

```
extremalindex(FCwx$MxT, 90)

fcTmax <- decluster(FCwx$MxT, 90,
    type = "runs", r = 12)

fcTmax

plot(fcTmax)

x <- c(fcTmax)

threshrange.plot(x, r=c(90, 95),
    type = “PP", nint = 20)
```

Introduction to EVA: Examples

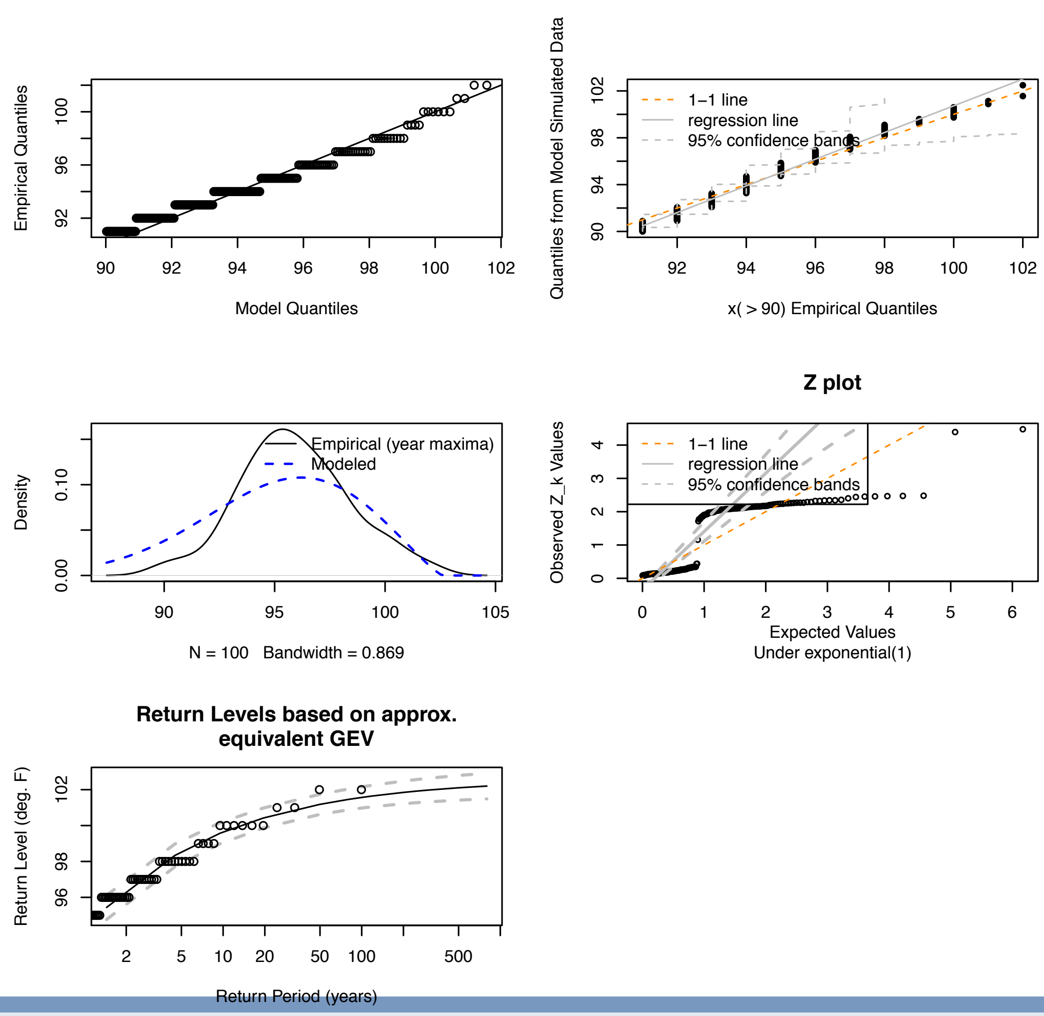
d) Point process

```
FCwx2 <- data.frame(x = x,
    year = FCwx$Year,
    month = FCwx$Mn,
    day = FCwx$Dy,
    doy = 1:length(x))

fit0 <- fevd(x, data = FCwx2,
    threshold = 90, type = “PP",
    units = "deg. F")

fit0

plot(fit0)
```

Introduction to EVA: Examples

d) Point process

fevd(x = x, data = FCwx2, threshold = 90, type = "PP", units = "deg. F")



Introduction to EVA: Examples

d) Point process

```
fit1 <- fevd(x, data = FCwx2,
    threshold = 90,
    location.fun =
    ~cos(2 * pi * doy / 365.25) +
    sin(2 * pi * doy / 365.25),
    type = “PP", units = "deg. F")

fit1

plot(fit1)

lr.test(fit0, fit1)
```

Introduction to EVA: Examples

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d) Point process

```
fit2 <- fevd(x, data = FCwx2,
    threshold = 90,
    location.fun =
    ~cos(2 * pi * doy / 365.25) +
    sin(2 * pi * doy / 365.25),
    scale.fun =
    ~cos(2 * pi * doy / 365.25) +
    sin(2 * pi * doy / 365.25),
    use.phi = TRUE,
    type = “PP", units = "deg. F")

fit2
plot(fit2)
lr.test(fit1, fit2)
```

Enough already! Let's try it ourselves…

Questions?