Evaluating spatial quantitative precipitation forecasts in the form of binary images

Merging and Matching with the Baddeley Delta Metric

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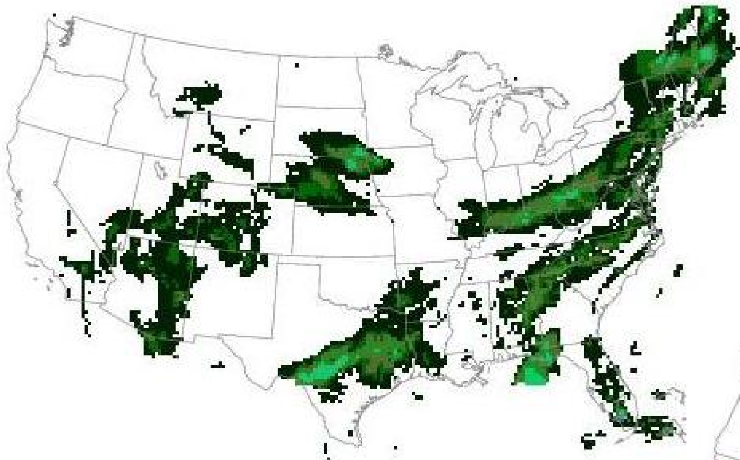
Outline

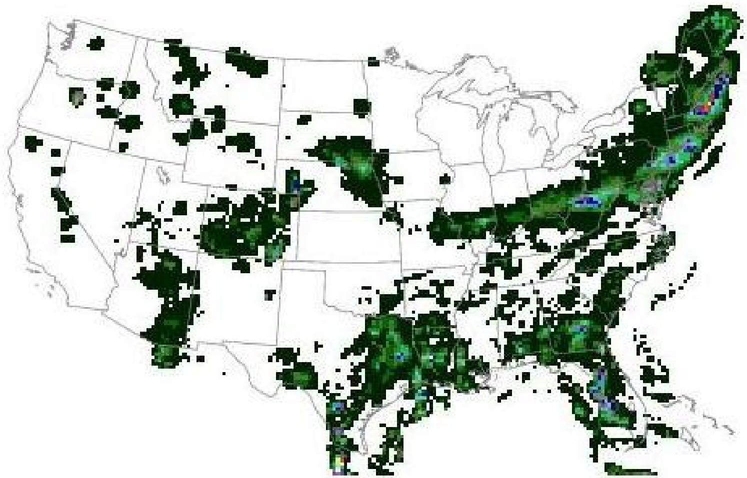
- Motivation
- Baddeley Delta Metric
- Merging and Matching Strategy
- Test Case Examples
- Summary and Ongoing Work

Motivation: Verifcation of Quantitative Precipitation Forecasts

Forecast

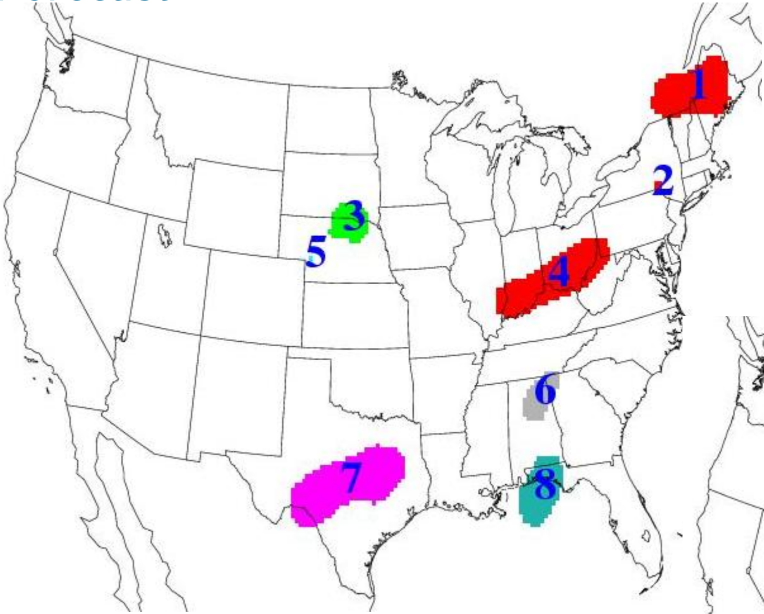


Observations

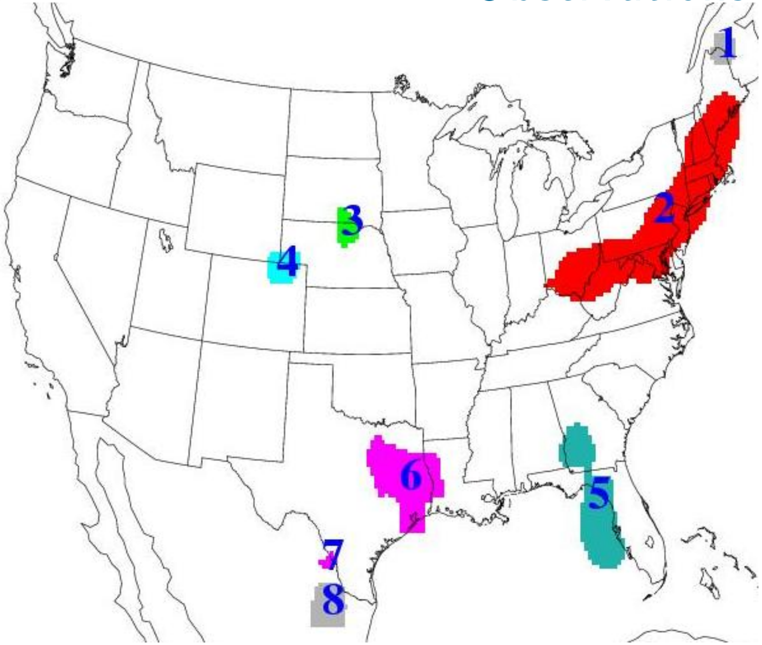


Motivation: Objectification Approach

Forecast

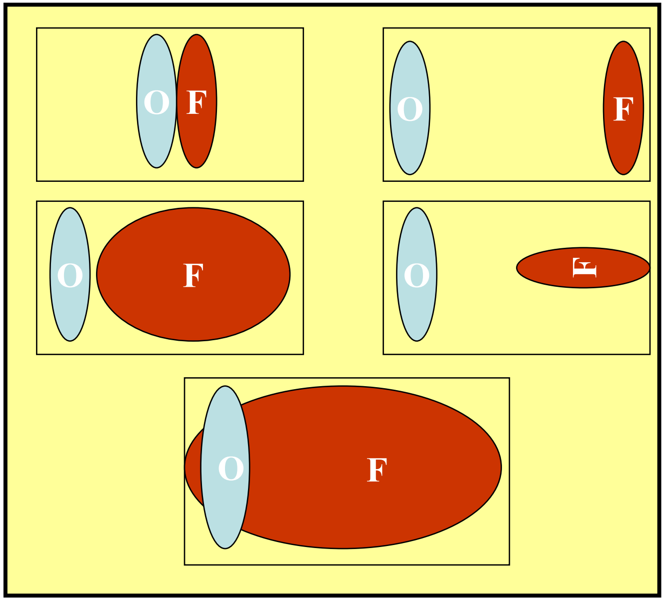


Observations



Motivation: Objectification Approach

Example



- First four forecasts have POD=0; FAR=1; CSI=0
  - i.e., all are equally "BAD"
- Fifth forecast has POD>0, FAR<1, CSI>1
- Traditional verification approach identifies "worst" forecast as the "best"

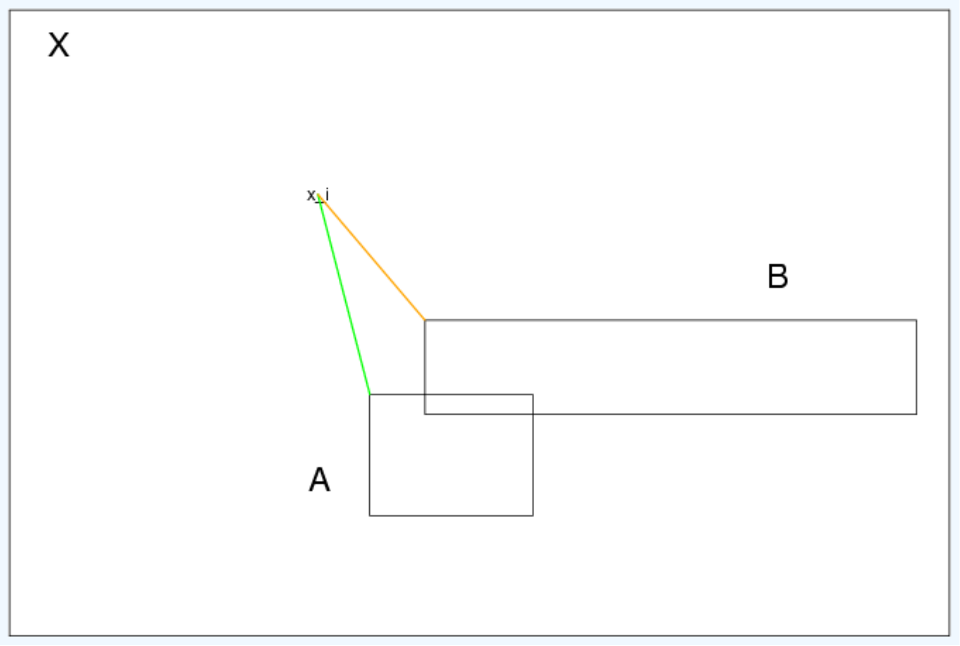
Object Comparisons Outline

The goal is to find the best mergings and matchings.

- We need a metric.
- Using the chosen metric, we need a reasonably fast strategy for merging and matching.
- Baddeley metric is designed for the purpose of comparing images, and it can be fast.

Baddeley Delta Metric

The Baddeley delta metric is essentially an average of shortest distances between *every* pixel in an image raster and a set.



Baddeley Delta Metric

For a raster of pixels, X, the Baddeley delta metric for comparing set $A \subseteq X$ to set $B \subseteq X$ ($\Delta_w^p(A, B)$) is:

$$\Delta_w^p(A, B) = \Delta = \left[\frac{1}{n(X)} \sum_{x \in X} |w(d(x, A)) - w(d(x, B))|^p \right]^{1/p},$$

where $d(x, A)$ is the shortest distance from a point $x \in X$ to the set (object) A, $1 \leq p < \infty$ and w is a concave function ($w(s + t) \leq w(s) + w(t)$) that is strictly increasing at zero ($w(t) = 0$ iff $t = 0$).

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We use $p = 2$ and $w(t) = \min(t, 100)$

Lower values of Δ mean sets are more similar to each other.

Merging and Matching Strategy

Given a forecast image object with n_f objects and an analysis image object with n_a objects.

- Which objects from one field match "best" with objects from the other field?
- Which objects within an image should be merged?
- Ideally, one would compute all $2^{n_f} \cdot 2^{n_a}$ Δ's for all possible mergings. **Too computationally intensive!**
- Here, we propose looking at a reasonable subset of the possible mergings.

Merging and Matching Strategy

Let $i = 1, \ldots, n_f$ denote the i^{th} forecast object, and $j = 1, \ldots, n_a$ the j^{th} analysis object.

1. Create the matrix $[\Delta(i, j)]$

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Let $i = 1, \ldots, n_f$ denote the i^{th} forecast object, and $j = 1, \ldots, n_a$ the j^{th} analysis object.

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2. Rank the values from Step 1.

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3. Create a matrix with $\Delta(i, j_{(1)}),$, $\Delta(i, j_{(1,\ 2)}), \ldots, \Delta(i, j_{(1,\ldots,n_a)})$ Do the same for the other direction. (i.e., $\Delta(j, i_{(1)}), \ldots, \Delta(j, i_{(1,\ldots,n_a)})$

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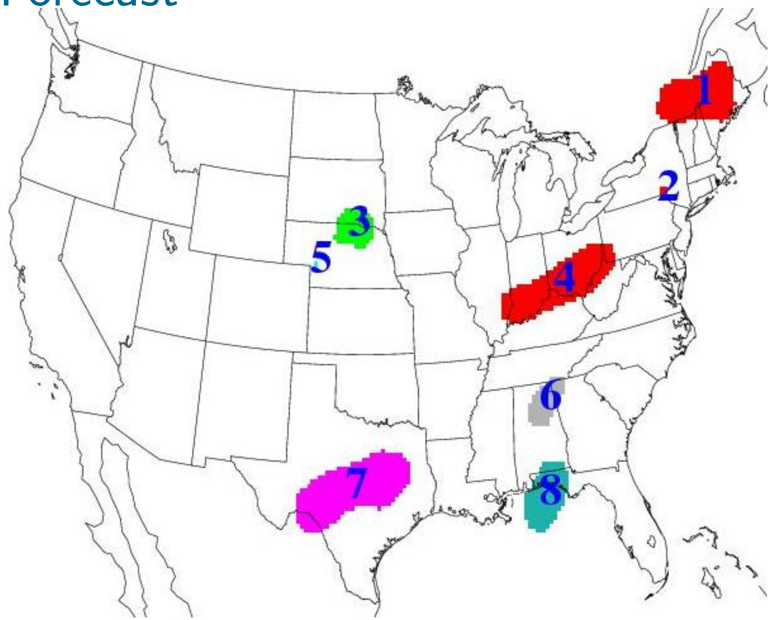
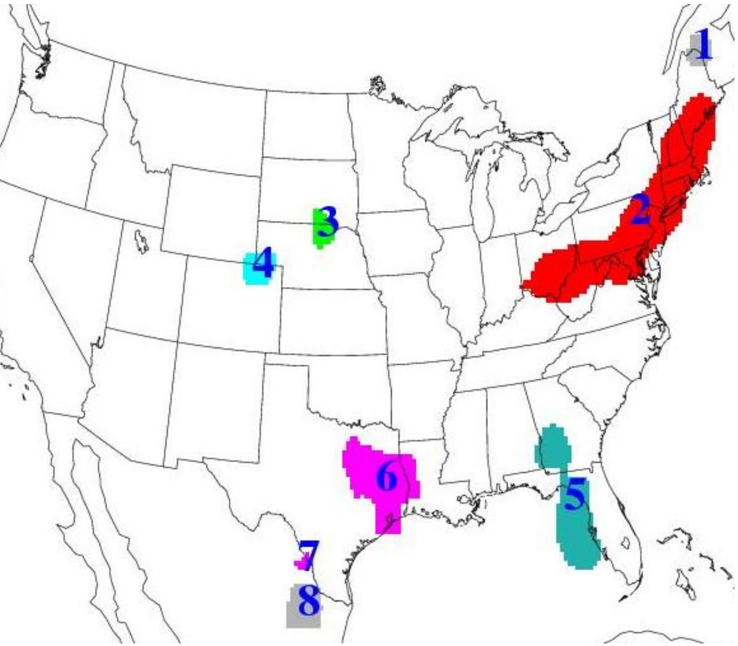
4. Merge and match objects by comparing the above three matrices.
5. Accept merges/matches only for Δ below a chosen threshold.

Test Case 1

Forecast	Observations	Δ
3	3	0.02
5	4	0.06
7	6, 7	0.07
1, 2, 4	2	0.08
8	5	0.08

Forecast

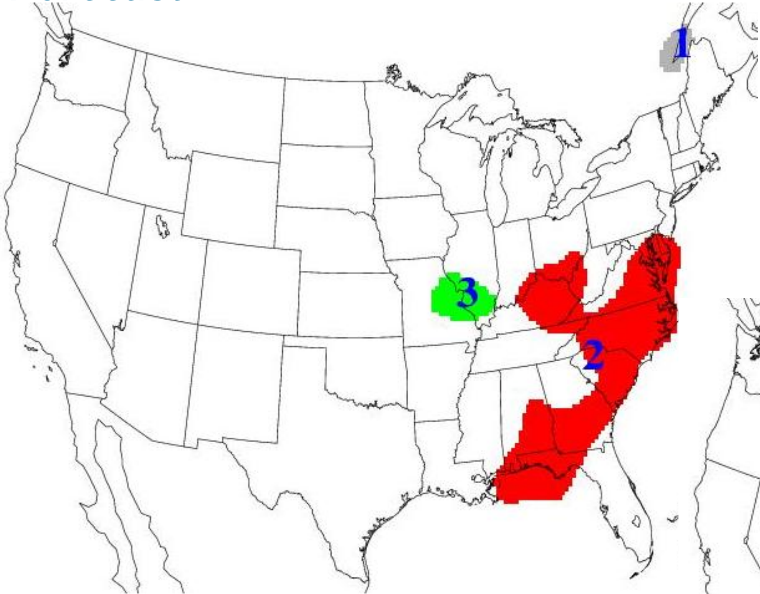




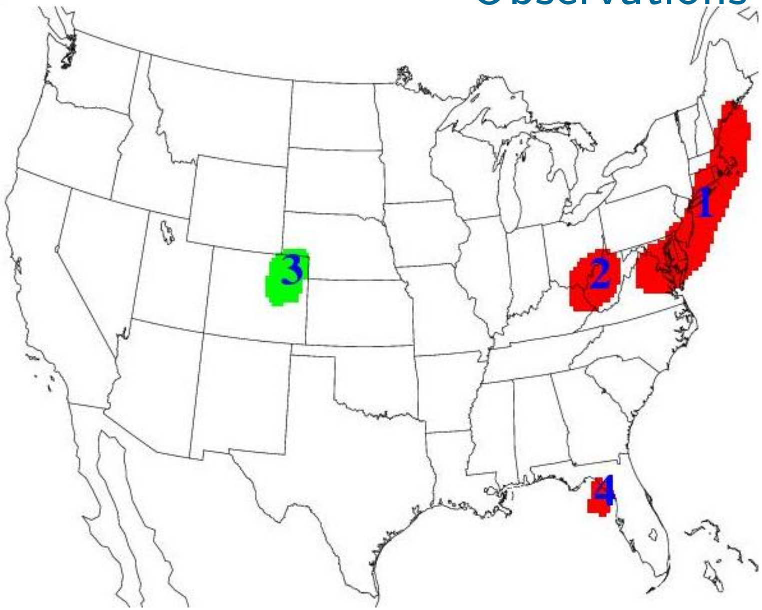
Observations

Test Case 2

Forecast



Observations



Forecast	Observations	$\triangle$
2	1, 2, 4	0.12
3	3	0.29

Summary and Ongoing Work

- Difficult to perform verification on QPF.
- One way to solve the problem is to objectify the QPF, and analyze the "cleaner" resulting objects.
- Before verification can be done on the resulting objects, they must be matched/merged.
- Baddeley delta metric is useful for comparing images.
- Need to compare our strategy with other approaches (e.g., fuzzy logic).
- Adapt our strategy so that the same (merged) analysis objects are compared to different forecasts.