VERIFICATION OF QUANTITATIVE PRECIPITATION FORECASTS USING BADDELEY'S DELTA METRIC

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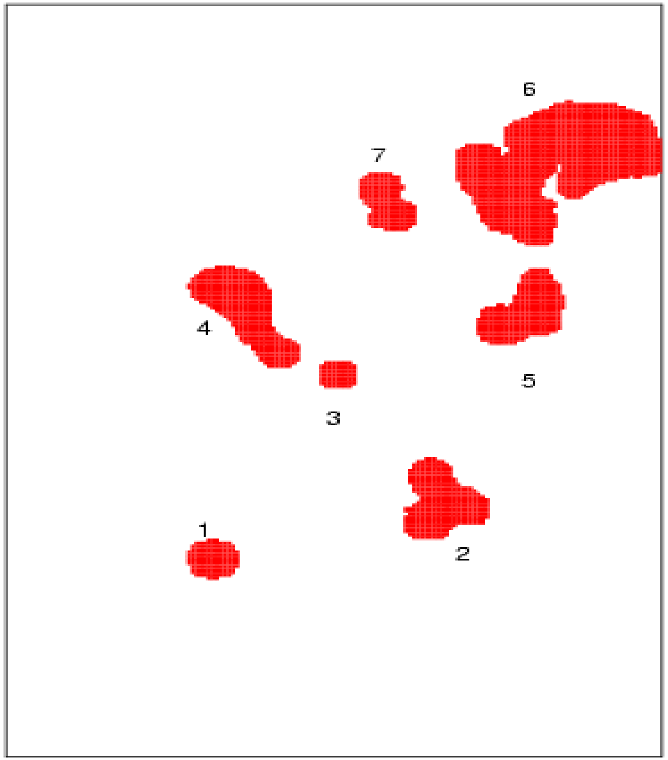
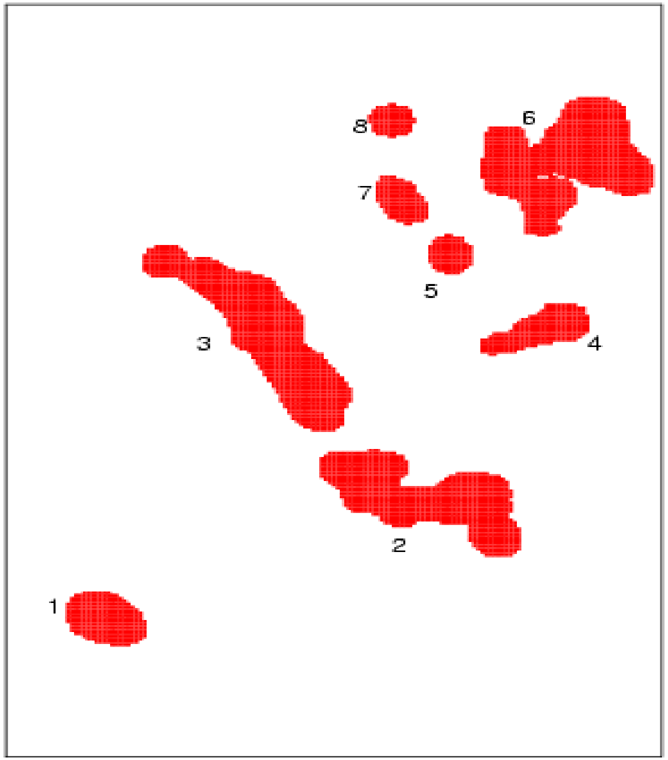
INTRODUCTION: BACKGROUND





INTRODUCTION: OUTLINE

- Localization performance measures
- Metrics and notation
- Baddeley's Delta Metric
- Using Baddeley's Delta Metric for object matching and merging
- Results from two image pairs
- Future and Ongoing Work

Localization performance measures

Some other choices of "distance" measures for comparing binary image objects A and B for a discrete raster X are: the mean error distance,

$$\bar{\mathrm{e}}(A, B) = \frac{1}{n(B)} \sum_{x \in B} d(x, A),$$

the mean square error distance

$$\bar{\mathrm{e}^2}(A, B) = \frac{1}{n(B)} \sum_{x \in B} d(x, A)^2,$$

among others.

Localization performance measures: some problems

Problems with the above error measures:

- Insensitive to type II errors (predicting an event when no event occurrs). For example, if $B \supseteq A$ (i.e., all errors are of type II), then $\bar{e} = \overline{e^2} = 0$ regardless of the positions of the type II errors.
- They are also insensitive to patterns of type I errors.

Metrics and notation

A metric is sensitive to type I and type II errors.

A metric, Δ, between two sets of pixels A and B contained in a pixel raster X satisfies the axioms

- $\Delta(A, B) = 0$ if and only if $A = B$;
- symmetry: $\Delta(A, B) = \Delta(B, A)$;
- triangle inequality: $\Delta(A, B) \leq \Delta(A, C) + \Delta(C, B)$

(Similarly, for the "distance" between two pixels x and y, say $\rho(x, y)$, in a raster of pixels. Just replace Δ with ρ and A, B with x, y.)

Metrics and notation

Let $d(x, A)$ denote the shortest distance from pixel x to $A \subseteq X$. That is,

$$d(x, A) = \min\{\rho(x, a) : \; a \in A\}$$

Also, $d(x, \emptyset) \equiv \infty$.

Metrics and notation

$d(\cdot, A)$ can be computed rapidly by the distance transform algorithm. See, for example:

Borgefors, G. Distance transformations in digital images. *Computer Vision, Graphics and Image Processing*, **34**:344–371, 1986.

Rosenfeld and Pfalz, J.L. Sequential operations in digital picture processing. *Journal of the Association for Computing Machinery*, **13**:471, 1966.

Rosenfeld and Pfalz, J.L. Distance functions on digital pictures. *Pattern Recognition*, **1**:33–61, 1968.

Metrics and notation: Hausdorff metric

Let $A, B \in X$, where X is a raster of pixels. The Hausdorff distance is given by:

$$H(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{x \in B} d(x, A)\}$$

That is, $H(A, B)$ is the maximum distance from a point in one set to the nearest point in the other set.

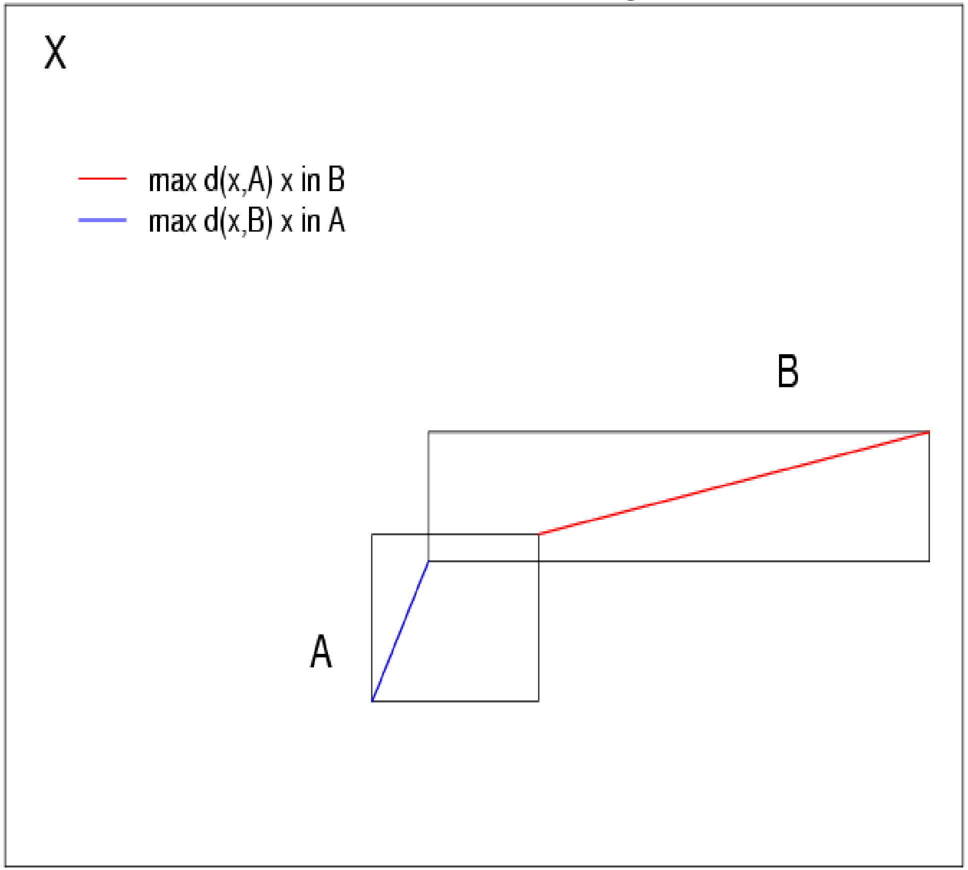
(Also set $H(\emptyset, \emptyset) = 0$ and $H(\emptyset, B) = H(B, \emptyset) = \infty$ for $B \neq \emptyset$)

Metrics and notation: Hausdorff metric

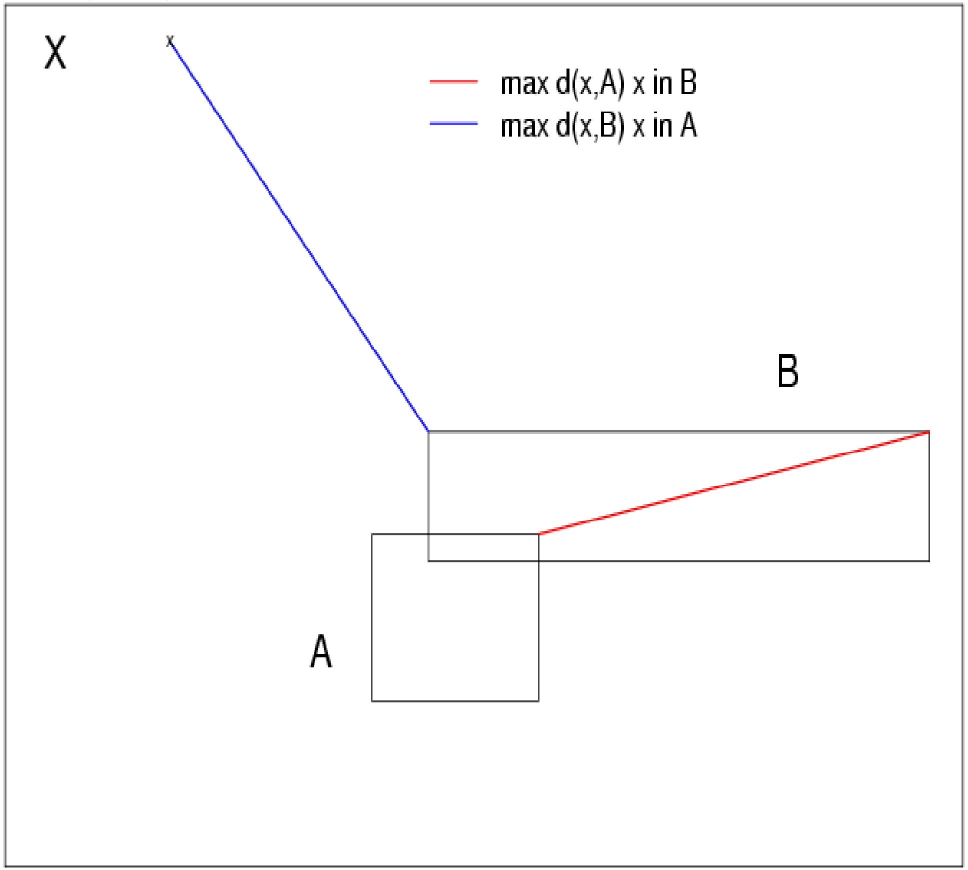
Under certain conditions (which are met for our purposes provided $A, B \neq \emptyset$) H can be written as:

$$H(A, B) = \sup_{x \in X} |d(x, A) - d(x, B)|$$

The Hausdorff metric is the length of the red line here.



H(A,B) has an extreme sensitivity to changes in even a small number of pixels.



Baddeley's Delta Metric

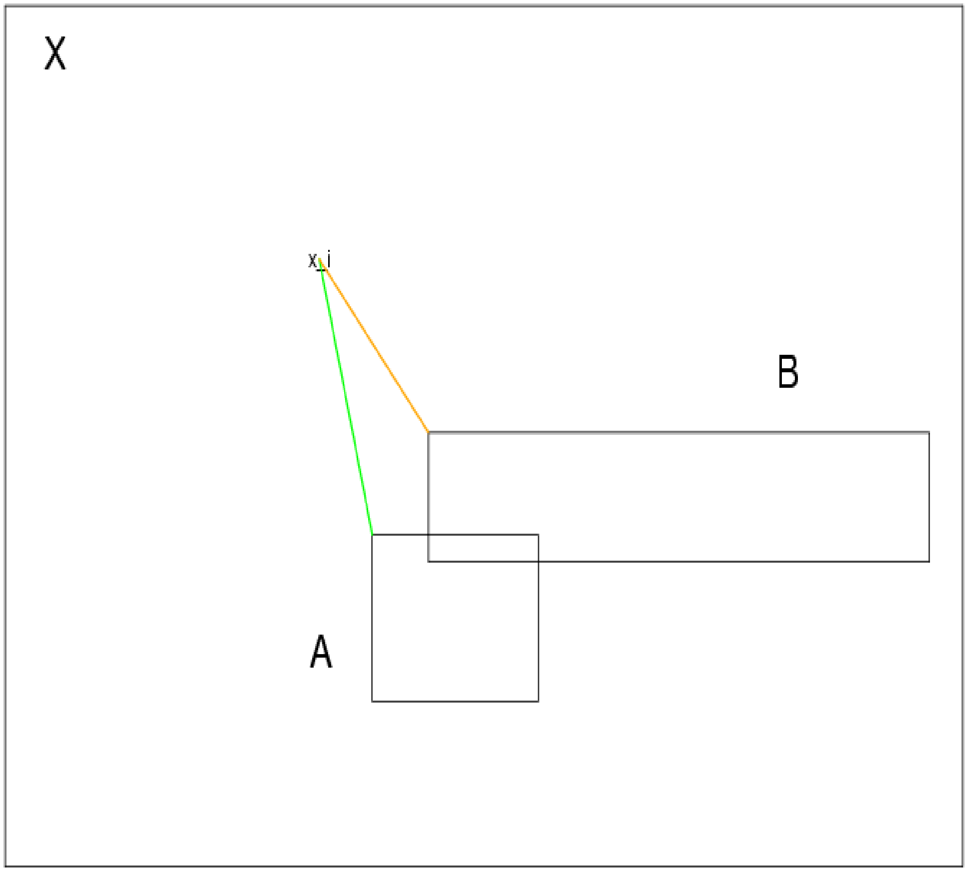
Replace the suprema in $H(A, B) = \sup_{x \in X} |d(x, A) - d(x, B)|$ with an L_p norm. That is,

$$\Delta^p(A, B) = \left[\frac{1}{n(X)} \sum_{x \in X} |d(x, A) - d(x, B)|^p \right]^{1/p}$$

for $1 \leq p < \infty$.

However, the above is not a bounded metric. One can simply transform the metrics $d(x, \cdot)$ so that it is a bounded metric. Specifically, let w be a concave function ($w(s + t) \leq w(s) + w(t)$) that is strictly increasing at zero ($w(t) = 0$ iff $t = 0$). The transformation used here is $w(t) = \min\{t, c\}$, for a fixed $c > 0$. So, the new metric, called Baddeley's delta, is given by

$$\Delta_w^p(A, B) = \left[\frac{1}{n(X)} \sum_{x \in X} |w(d(x, A)) - w(d(x, B))|^p \right]^{1/p}$$


X
x_i
B
A

Using Baddeley's Delta Metric for object matching and merging

Given a forecast image object with n_f objects and an analysis image object with n_a objects.

- Which objects from one field match "best" with objects from the other field.
- Which objects within an image should be merged?
- Ideally, one would compute Δ for all possible mergings. However, there are $2^{n_f} \cdot 2^{n_a}$ possible mergings; which would generally be too computationally intensive to be compared in practice.
- Here, we propose looking at a reasonable subset of the possible mergings.

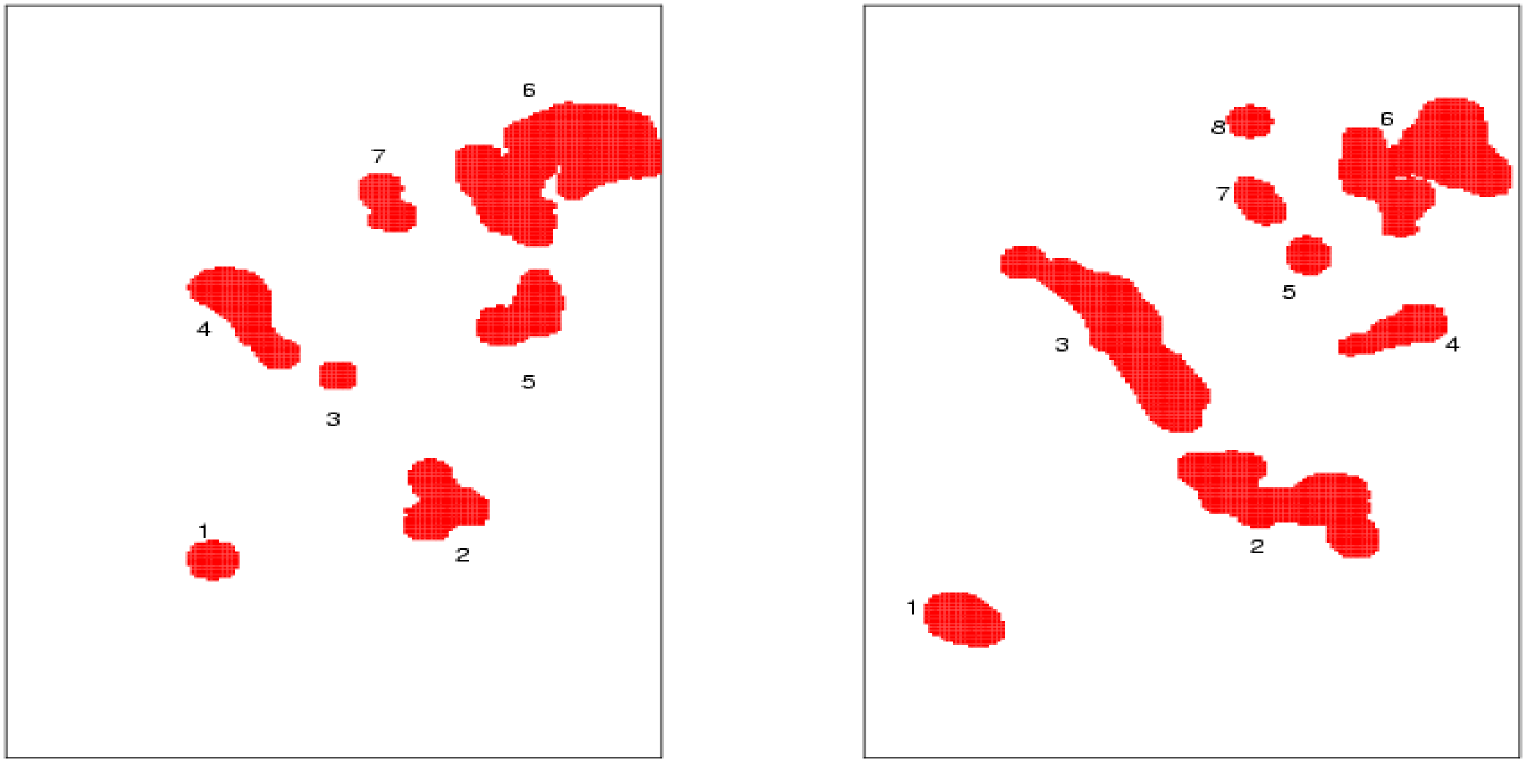
Using Baddeley's Delta Metric for object matching and merging

The proposed technique is as follows.

Let $i = 1, \dots, n_f$ denote the i^{th} forecast object, and $j = 1, \dots, n_a$ the j^{th} analysis object.

1. Compute Δ for each object from forecast with each object from analysis.
2. Rank the values from Step 1. For the i^{th} forecast image, let $j_1, \dots, j_{n_a}$ denote the lowest to highest delta between object i and each object j. Similarly for the j^{th} analysis object denote $i_1, \dots, i_{n_f}$ as the lowest to highest delta when comparing object j to each forecast object.
3. Compute Δ between the i^{th} forecast object and object j_1, then between i and j_1 and j_2 (merged together), and so on until object i is compared to the merging of all n_a objects from the analysis image.
4. Perform Steps 3 and 4 in the other direction. That is, compute the delta between object j and i_1, j and i_1 and i_2, etc ...
5. Merge and match objects by comparing the above three Baddeley scores.

Results from two image pairs



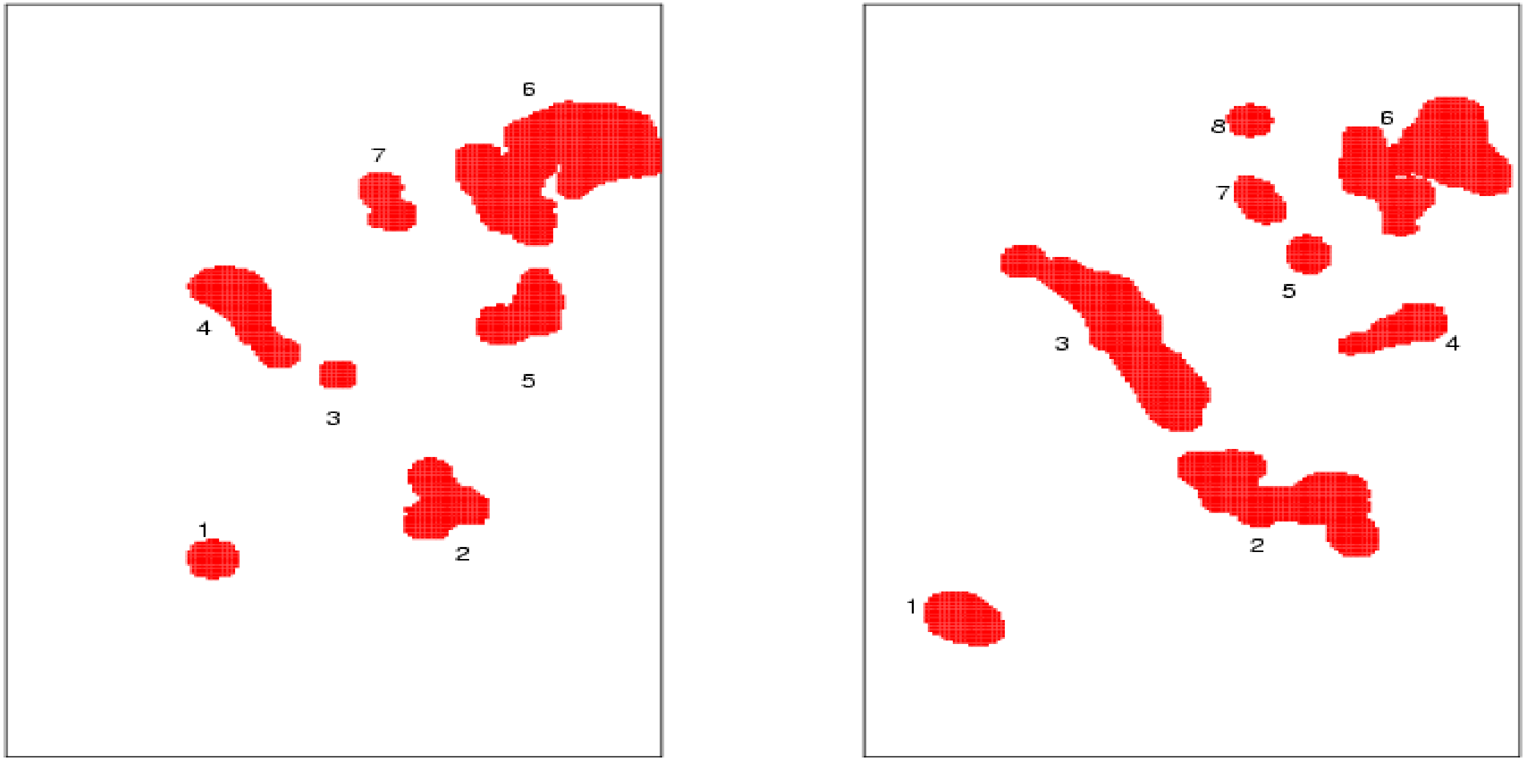
First image pair example: left is analysis image and right is forecast image. Overall Baddeley delta metric is about 0.271.

One-to-one Baddeley scores for each image (rows are analysis objects and columns are forecast objects).

	1	2	3	4	5	6	7	8
1	0.156	0.328	0.440	0.461	0.487	0.542	0.509	0.505
2	0.415	0.099	0.391	0.331	0.426	0.510	0.482	0.515
3	0.428	0.295	**0.219**	0.241	0.274	0.411	0.334	0.410
4	0.483	0.407	**0.119**	0.357	0.317	0.444	0.322	0.394
5	0.500	0.402	0.389	**0.054**	0.144	0.249	0.255	0.348
6	0.547	0.553	0.482	0.290	0.176	**0.033**	0.169	0.212
7	0.514	0.518	0.369	0.296	0.136	0.201	**0.027**	0.154

Ranks from above matrix.

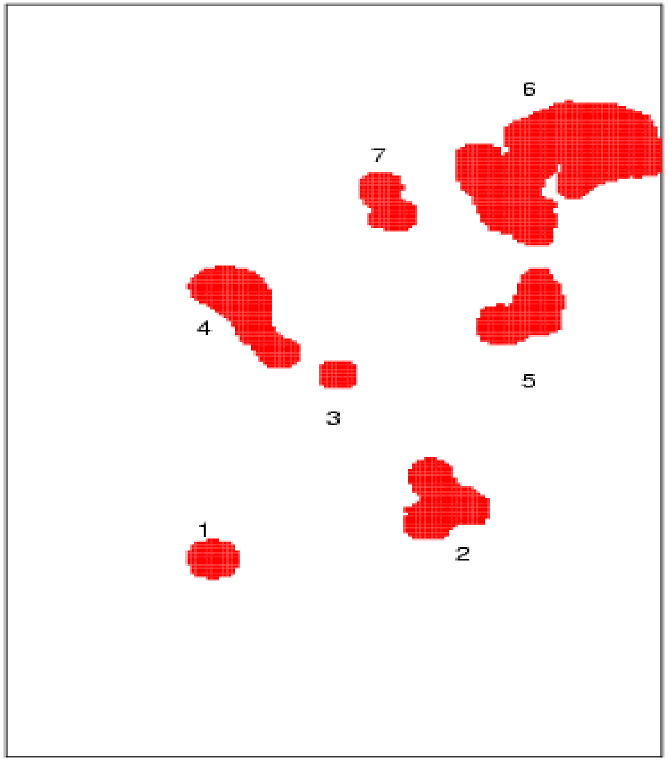
	1	2	3	4	5	6	7	8
1	9	24	40	42	46	54	49	48
2	37	4	31	25	38	50	44	52
3	39	20	14	15	18	36	26	35
4	45	34	5	28	22	41	23	32
5	47	33	30	**3**	7	16	17	27
6	55	56	43	19	11	**2**	10	13
7	51	53	29	21	6	12	**1**	8

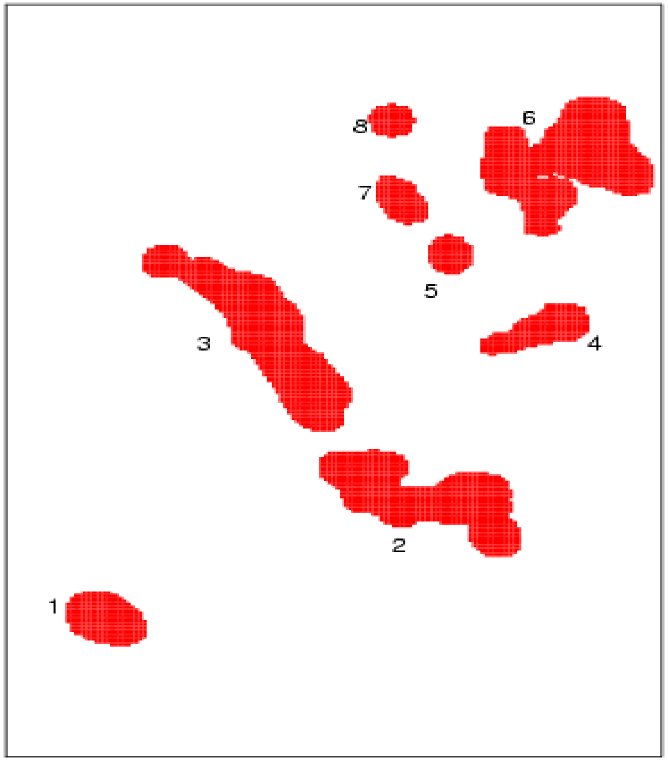

6
7
4
3
5
1
2
8
6
7
5
3
4
2
1

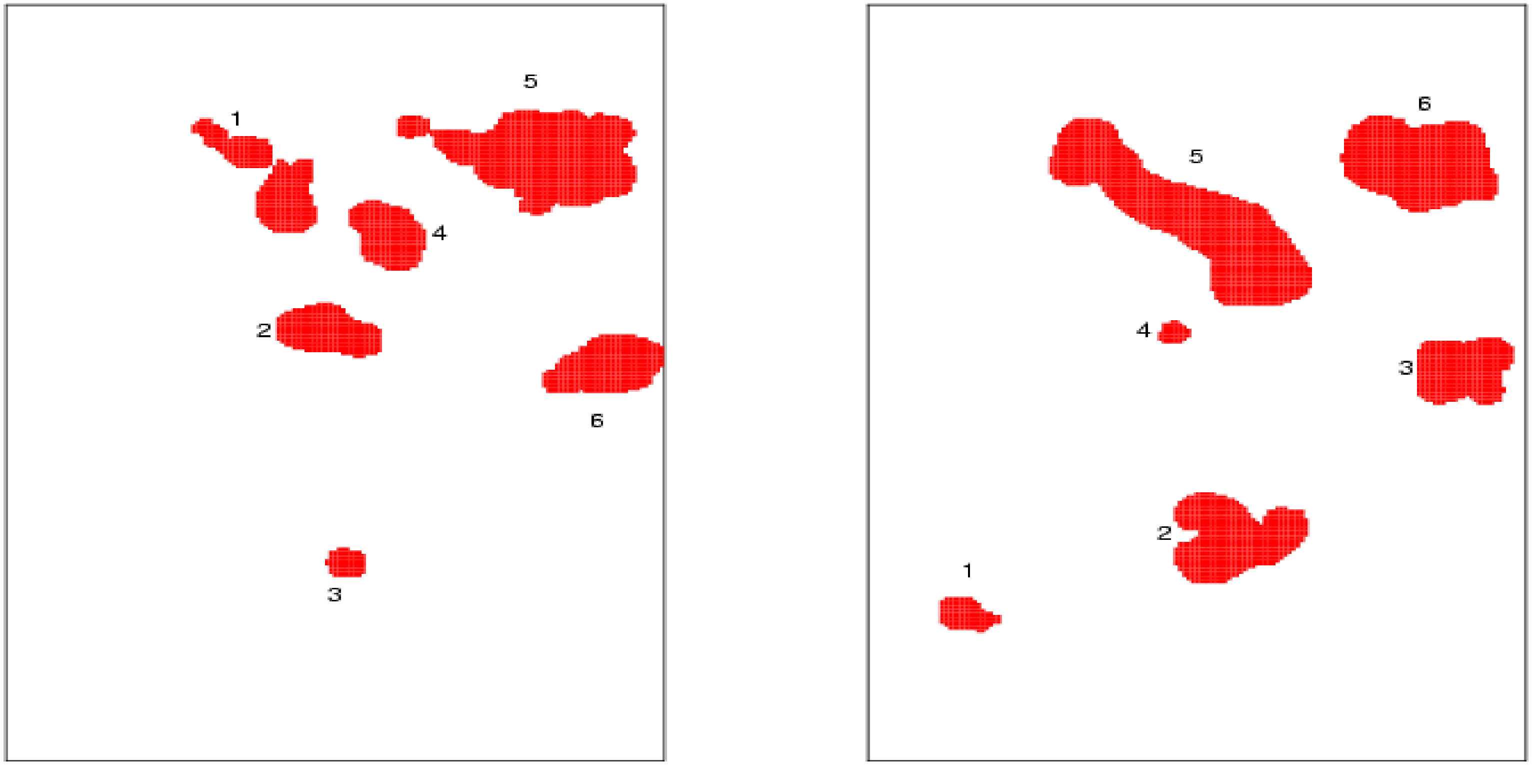
Results

Results of analysis-forecast matching. Note: forecast objects 5 and 8 not matched to any analysis object.

Analysis (A)	Forecast (B)	Baddeley delta score
7	7	0.027
6	6	0.033
5	4	0.054
4 and 3	3	0.080
2	2	0.099
1	1	0.156


6
7
4
5
3
1
2


8
6
7
5
3
4
2
1



Second image pair example: left is analysis image and right is forecast image. Overall Baddeley Delta metric is about 0.223.

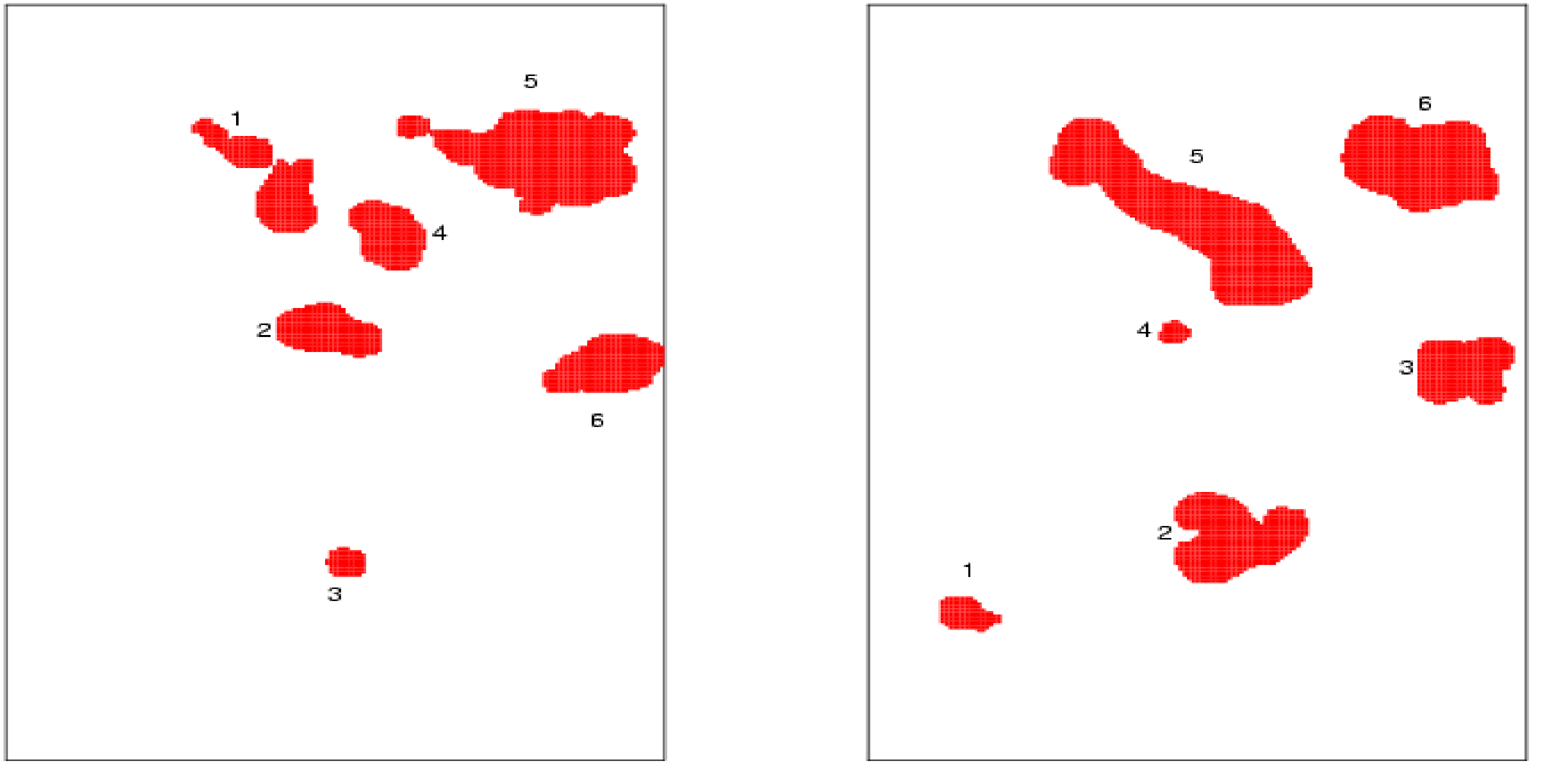
Some Results

Baddeley delta one-to-one comparisons. Rows are analysis objects and columns are forecast objects.

	1	2	3	4	5	6
1	0.533	0.567	0.454	0.303	**0.170**	0.346
2	0.475	0.407	0.313	**0.071**	0.278	0.391
3	0.281	0.117	0.393	0.395	0.570	0.525
4	0.512	0.515	0.331	0.229	**0.177**	0.229
5	0.543	0.587	0.384	0.398	0.307	**0.081**
6	0.462	0.391	**0.015**	0.311	0.422	0.346

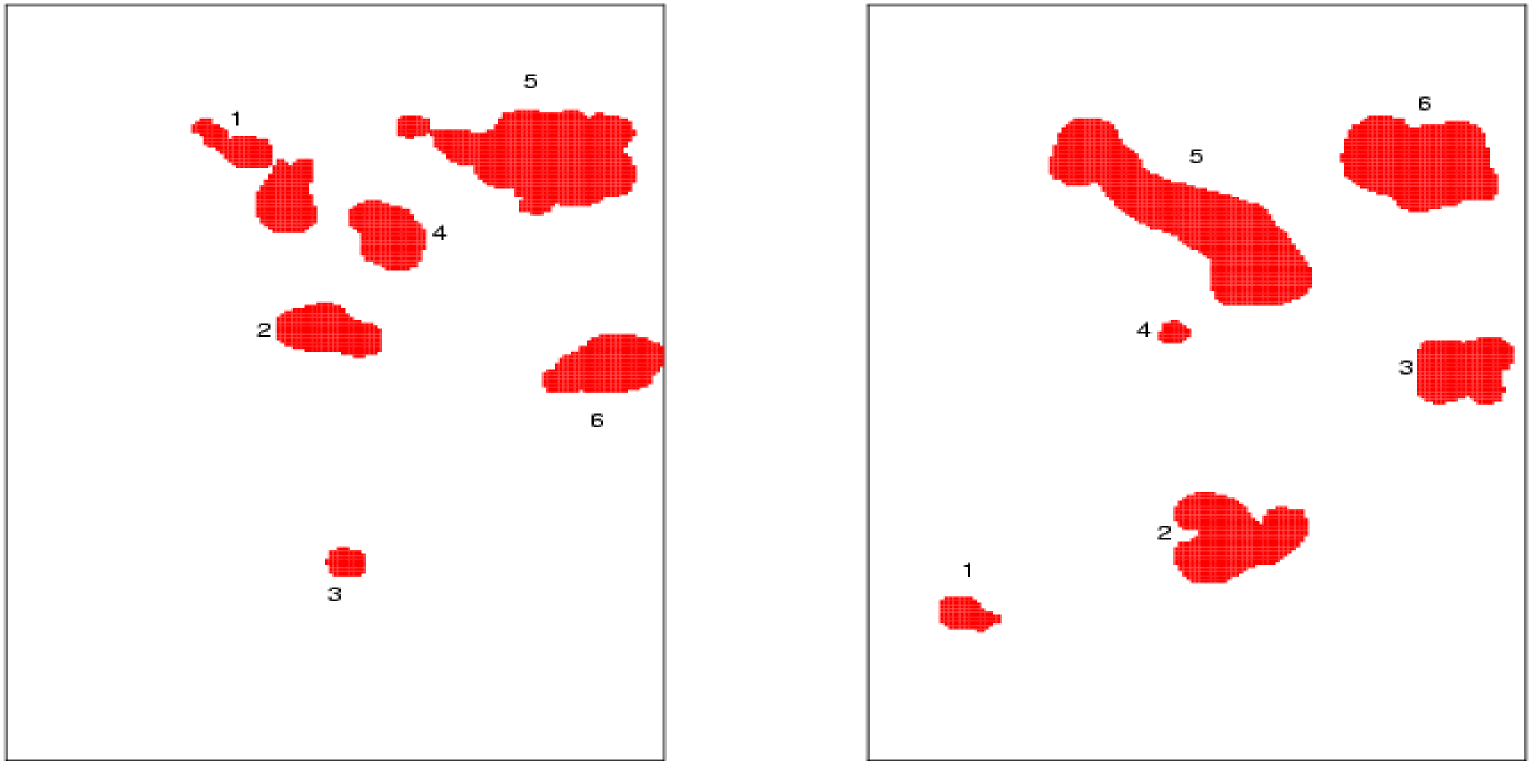
Ranks from previous matrix.

	1	2	3	4	5	6
1	32	34	26	11	5	16
2	28	24	14	2	9	19
3	10	4	21	22	35	31
4	29	30	15	8	6	7
5	33	36	18	23	12	3
6	27	20	1	13	25	17


1
2
3
4
5
6
1
2
3
4
5
6

Results of matching analysis objects to forecast objects. Note: forecast object 1 not matched.

Analysis	Forecast	Baddeley delta score
6	3	0.015
1 and 4	5	0.069
2	4	0.071
5	6	0.081
3	2	0.117


5
1
4
2
6
3
6
5
4
3
2
1

Future and Ongoing Work

- How to combine information from “best” matches/merges to give a summary score based on the Baddeley delta.
- What constitutes a “good” Baddeley delta score (have a human expert judge several cases?).
- How to incorporate into overall verification scheme.
- Characteristics/distributions of Δ’s.
- How to compare with Fuzzy Logic analysis of Bullock et al.

References

Baddeley, A.J., 1992: Errors in binary images and an L^p version of the Hausdorff metric, *Nieuw Archief voor Wiskunde*, **10**: 157–183.

Baddeley, A.J., 1992: An error metric for binary images, In W. Forstner and S. Ruwiedel (ed.) *Robust Computer Vision Algorithms*, Proceedings, International Workshop on Robust Computer Vision, Bonn. Karlsruhe: Wichmann, 59–78.

See: `http://www.maths.uwa.edu.au/~adrian/metrics.html`