NCAR

A spatial prediction comparison test for competing models

RAL Retreat, 8 December 2014

Center Green 1 Auditorium: Center

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Joint Numerical Testbed

National Center for Atmospheric Research

Univariate Setting

Diebold-Mariano test

Consider three series:

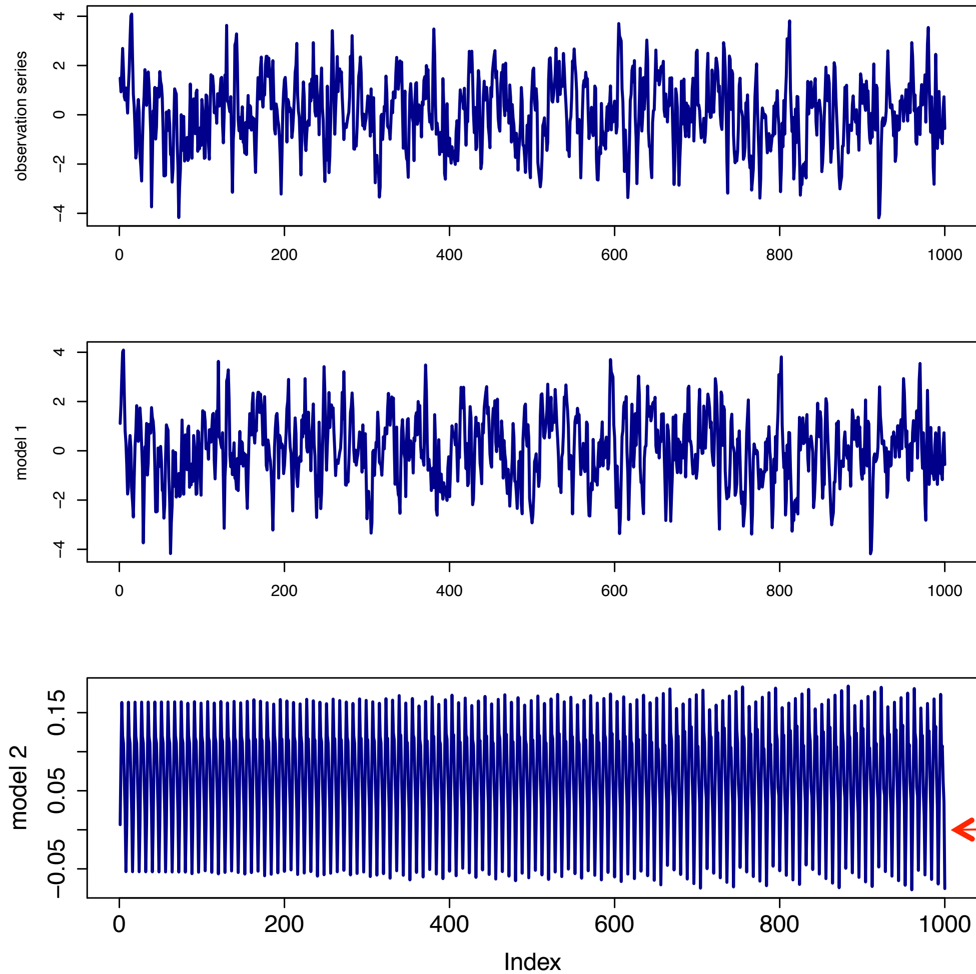
0. Observation: AR(2) time series given by
   $X_t = 0.8 * X_{t-1} - 0.2 * X_{t-2}$

1. Model 1: Same series as above, but shifted ten places to the left.
   $[\text{model 1}]_t = [\text{Observation}]_{t+10}$

2. Model 2: A smooth Fourier series fit to the observations in 0 above.
   $[\text{model 2}] \approx 0.07 - 0.01 * \cos(2\pi * t / 2) + \sin(2\pi * t / 2) + \ldots + \sin(2\pi * t / 8)$

Univariate Setting

Diebold-Mariano test



Simulated AR(2) series

Above shifted 10 places to the left

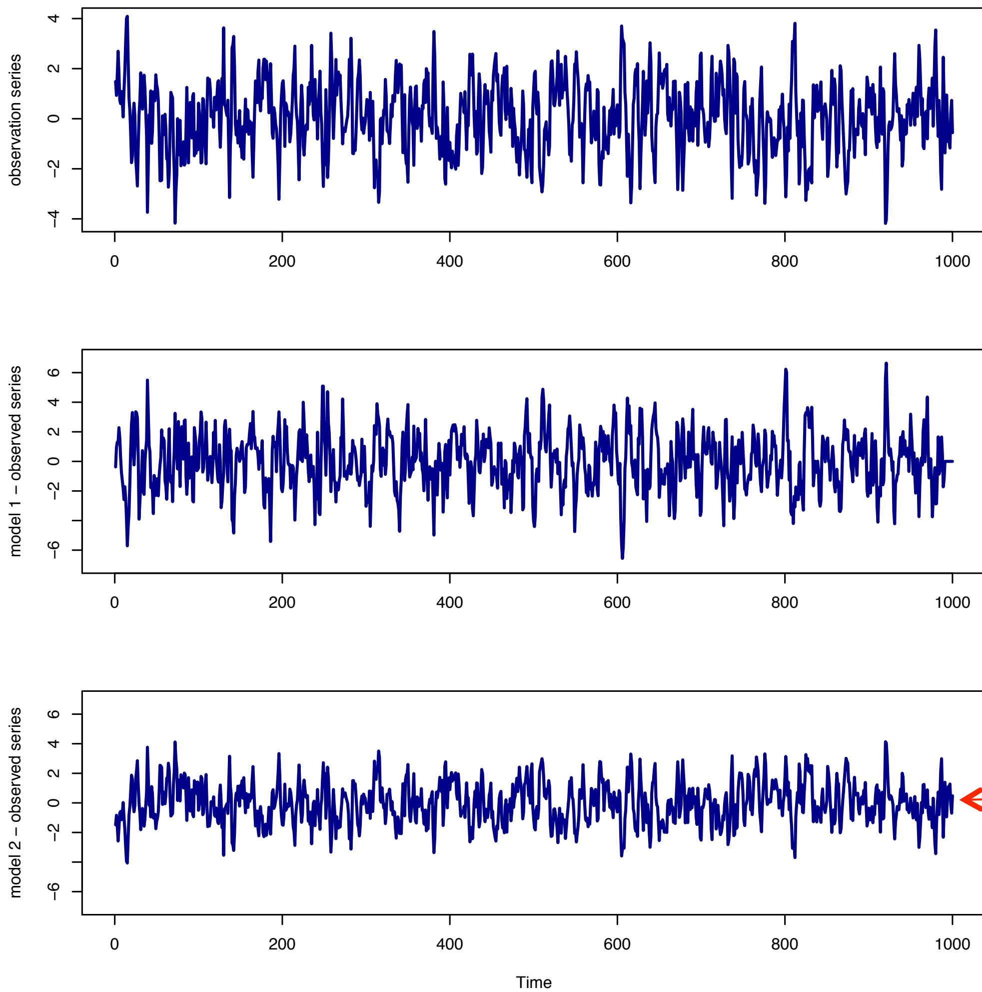
Fourier series fit to observed series

Univariate Setting

Diebold-Mariano test

Simple loss

F - O



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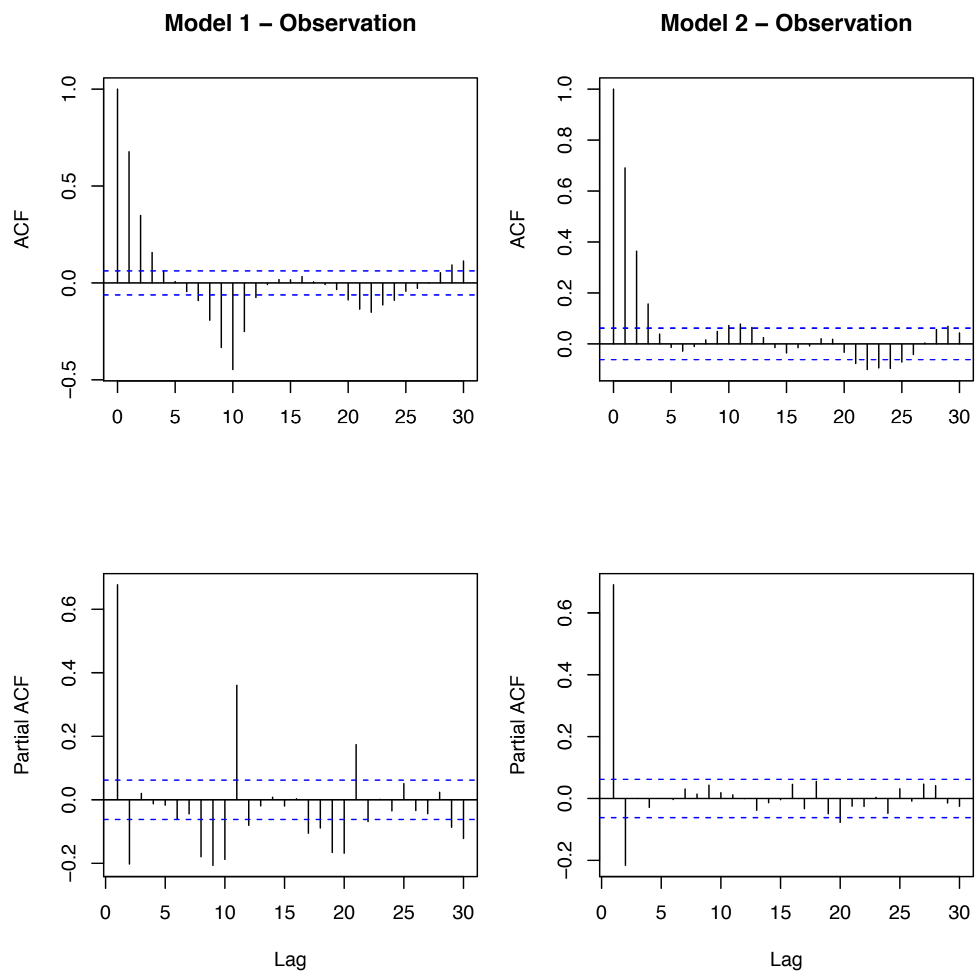


Univariate Setting

Diebold-Mariano test

- Let $\mathbf{x} = x_1, \ldots, x_n$ be an observed time series.
- Let $\mathbf{y} = y_1, \ldots, y_n$ and $\mathbf{z} = z_1, \ldots, z_n$ be two competing forecast models for **x**.
- Let **g**(**x**, **y**) and **g**(**x**, **z**) be the loss (or skill) function between the modeled and observed time series (defined at each time point!).
- Null hypothesis of interest is:

$$H_0: E[\mathbf{g}(\mathbf{x}, \mathbf{y})] = E[\mathbf{g}(\mathbf{x}, \mathbf{z})]$$

- Interest is in the "loss differential"

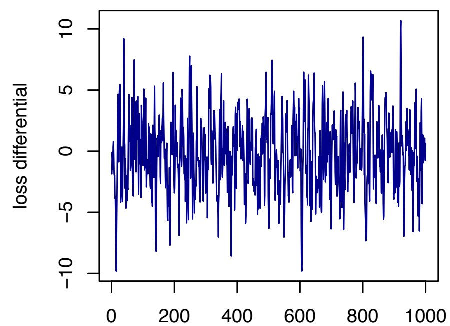
$$\mathbf{d} = \mathbf{g}(\mathbf{x}, \mathbf{y}) - \mathbf{g}(\mathbf{x}, \mathbf{z})$$

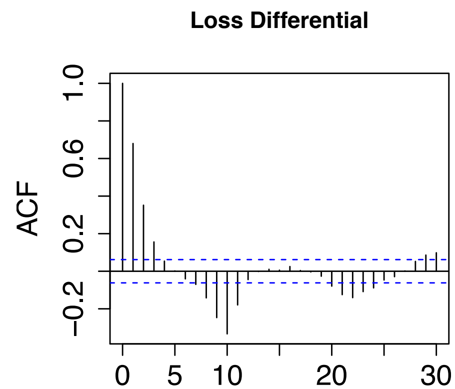
OR

$$H_0: E[d_t] = 0$$

Univariate Setting

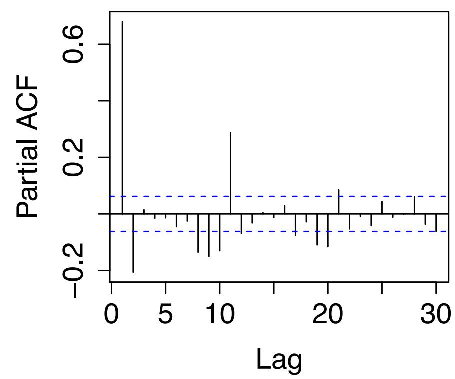
Diebold-Mariano test





Simple loss for these series:
mean(**d**) ≈ -0.2

Absolute error loss for these series:
mean(**d**) ≈ 7.5



Univariate Setting

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Diebold-Mariano test

Test Statistic:

$S = (\text{mean}(\mathbf{d}) - \mu_d) / (2\pi * s_d(0))$

Interest is generally in $\mu_d = 0$.

Key is in estimating $s_d(0)$

Obtained through a weighted sum of sample autocovariances (Diebold and Mariano, 1995, *J. Bus. Econ. Stat.*, **13**: 253—263)

Hering and Genton (2011, *Technometrics*, **53**, (4): 414—425) suggest fitting a parametric autocovariance model to the sample autocovariances first.

Univariate Setting

Diebold-Mariano test

Test Statistic:

$S = (\text{mean}(\mathbf{d}) - \mu_d) / (2\pi * s_d(0))$

Key is in estimating $s_d(0)$

Assumption: $S \longrightarrow N(0,1)$ as $n \longrightarrow \infty$

Univariate Setting

Diebold-Mariano test

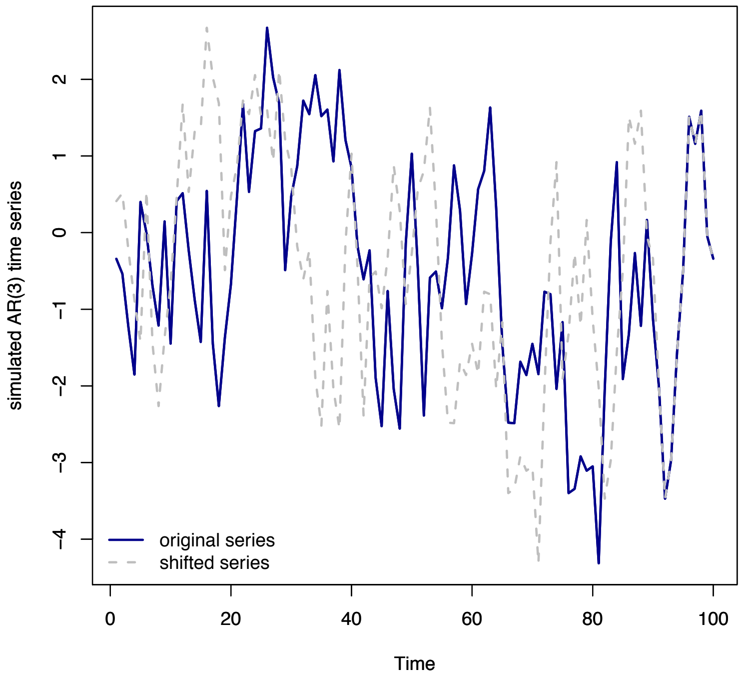
Our example:

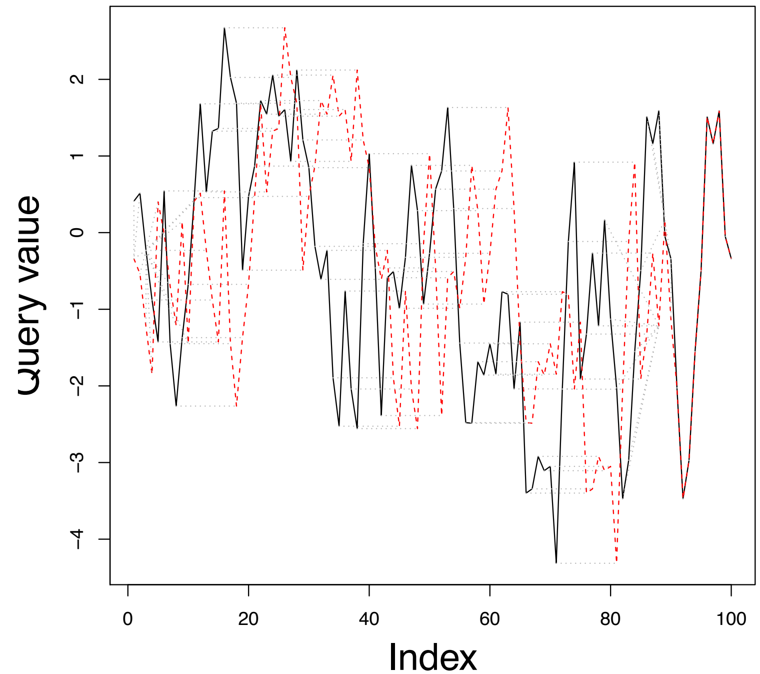
Simple loss: mean(**d**) ≈ -0.2 and p-value ≈ 0.8 (not significant)

Absolute Error loss: mean(**d**) ≈ 7.5 and p-value ≈ 0 (significant)

Univariate Setting

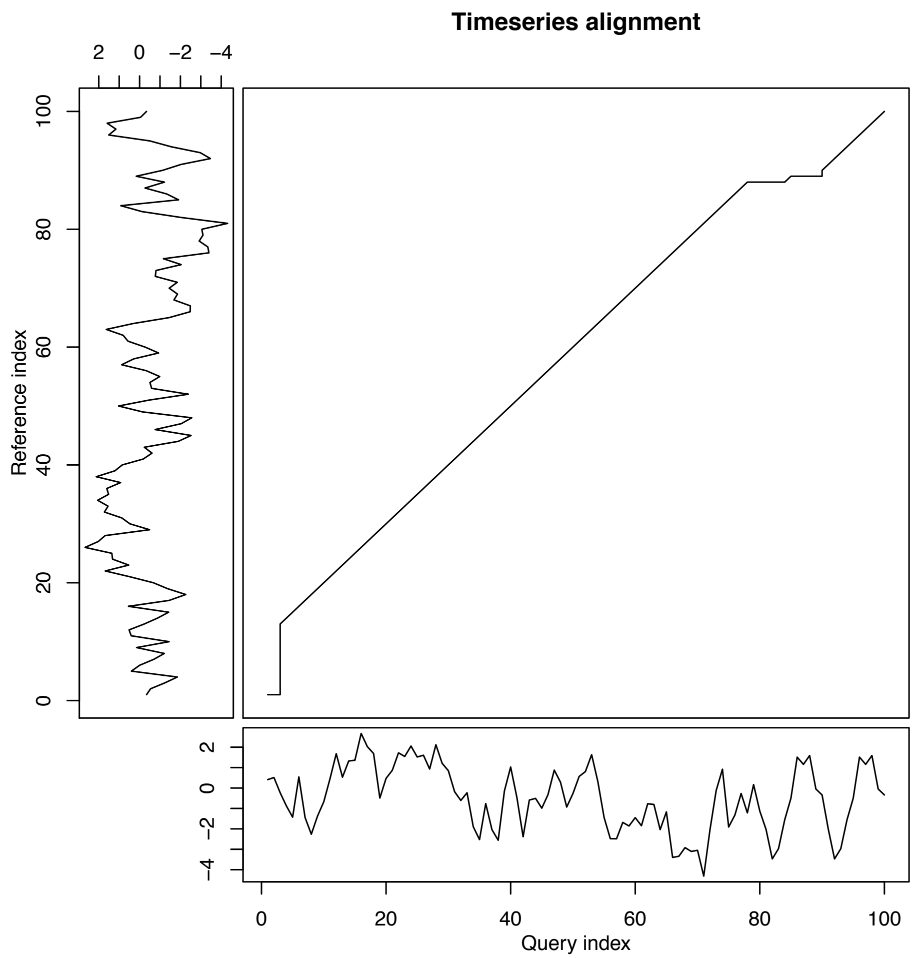
Dynamic Time Warping (DTW)





Univariate Setting

Dynamic Time Warping (DTW)



Univariate Setting

Dynamic Time Warping (DTW)

G. and Roux (2014, accepted to *Meteorol. Appl.*)

introduce loss function based on DTW:

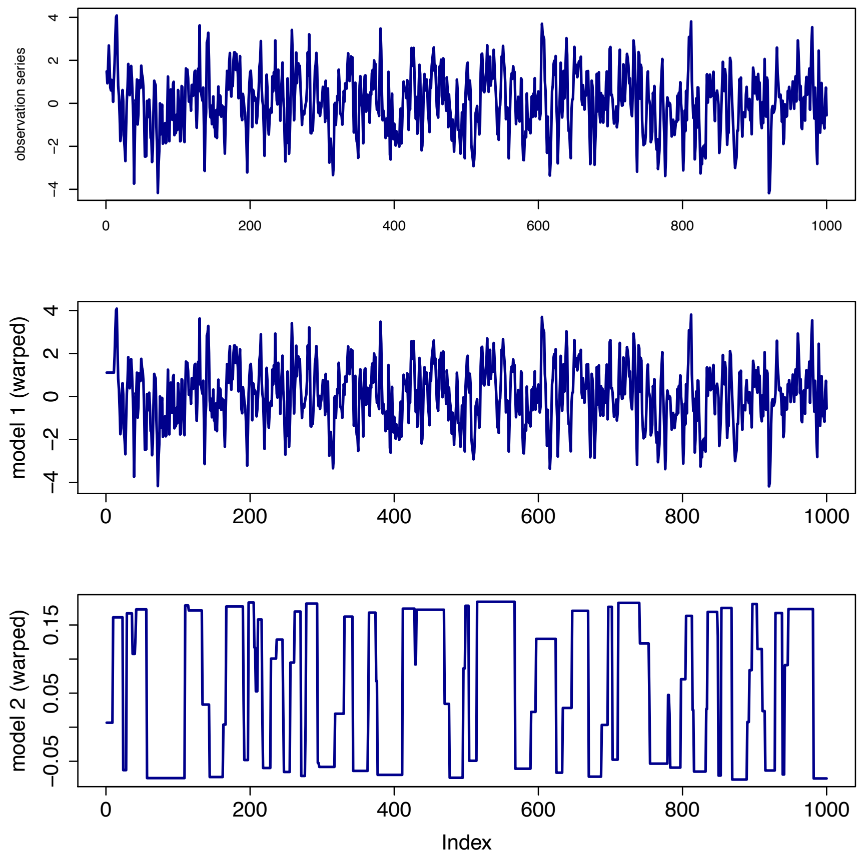
$$g(x_t, y_t) = f(t, w(t)) + h(x_t, y_{w(t)})$$

distance traveled in time → $f(t, w(t))$

Usual loss function → $h(x_t, y_{w(t)})$

Univariate Setting

Dynamic Time Warping (DTW)

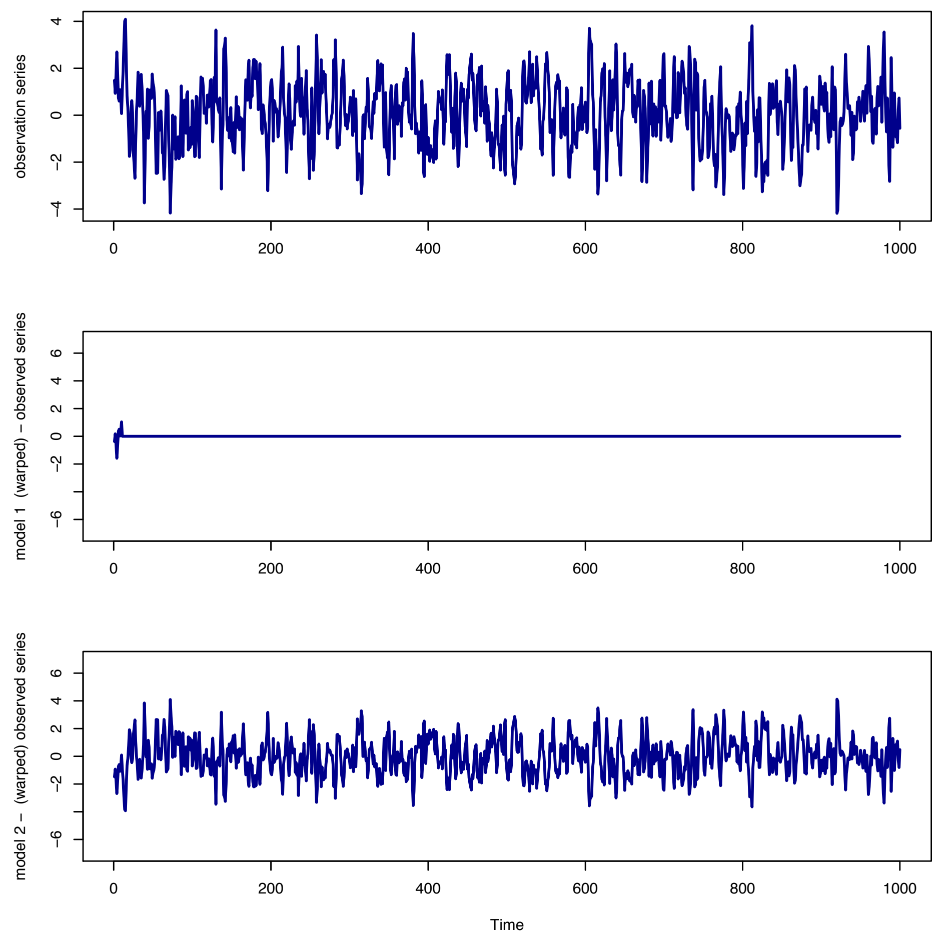


Univariate Setting

Dynamic Time Warping (DTW)

Absolute error loss:
mean(d) ≈ -0.97
p-value ≈ 0.17 (not significant)

Recall that without warping: Absolute error loss for these series:
mean(**d**) ≈ 7.5 and
p-value ≈ 0
(significant)



Univariate Setting

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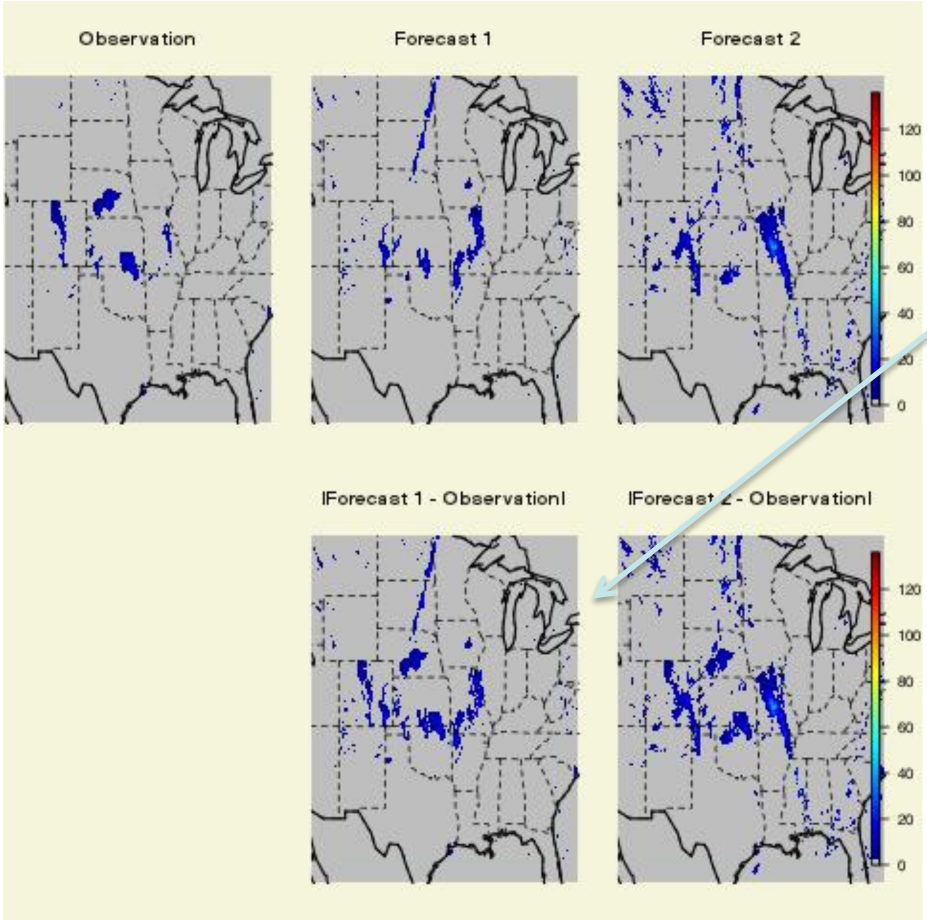
DM Test and Dynamic Time Warping (DTW)

Summary of Univariate Setting

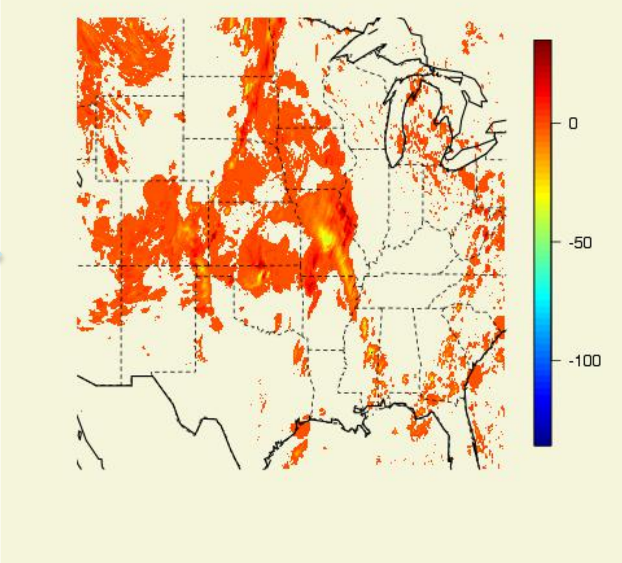
- Diebold-Mariano (DM) test gives an hypothesis test for competing forecasts (which forecast is better in terms of a loss (skill) function).
- Can also get confidence intervals instead of hypothesis test.
- Test accounts directly for temporal correlation.
- Robust to contemporaneous correlation (Hering and Genton, 2011).
- Works for any loss/skill function.
- No distributional assumptions for underlying series (only on the mean of the loss differential).
- powerful test (Hering and Genton, 2011).
- Dynamic Time Warping (DTW) allows for analyzing forecast performance while accounting for timing errors.
- `R` software package `verification`

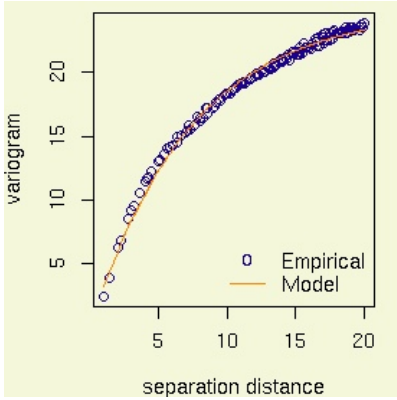
Spatial Prediction Comparison Test



D1 D2

D = D1 – D2





Spatial Prediction Comparison Test

Introduced by Hering and Genton (2011)

Extension of the time series version introduced by Diebold and Mariano (1995).

Spatial Prediction Comparison Test

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$$S = \frac{\bar{D}}{\sqrt{\operatorname{var}\left(\bar{D}\right)}} \longrightarrow \text{N}(0,\, 1) \text{ as } L \longrightarrow \infty, \text{ where}$$

$$\operatorname{var}\left(\bar{D}\right) = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} C\left(h_{ij}\right)$$

distance between two spatial locations

Covariance function for the loss differential's spatial dependence structure (need to replace with an estimate)

Accounting for Location Errors

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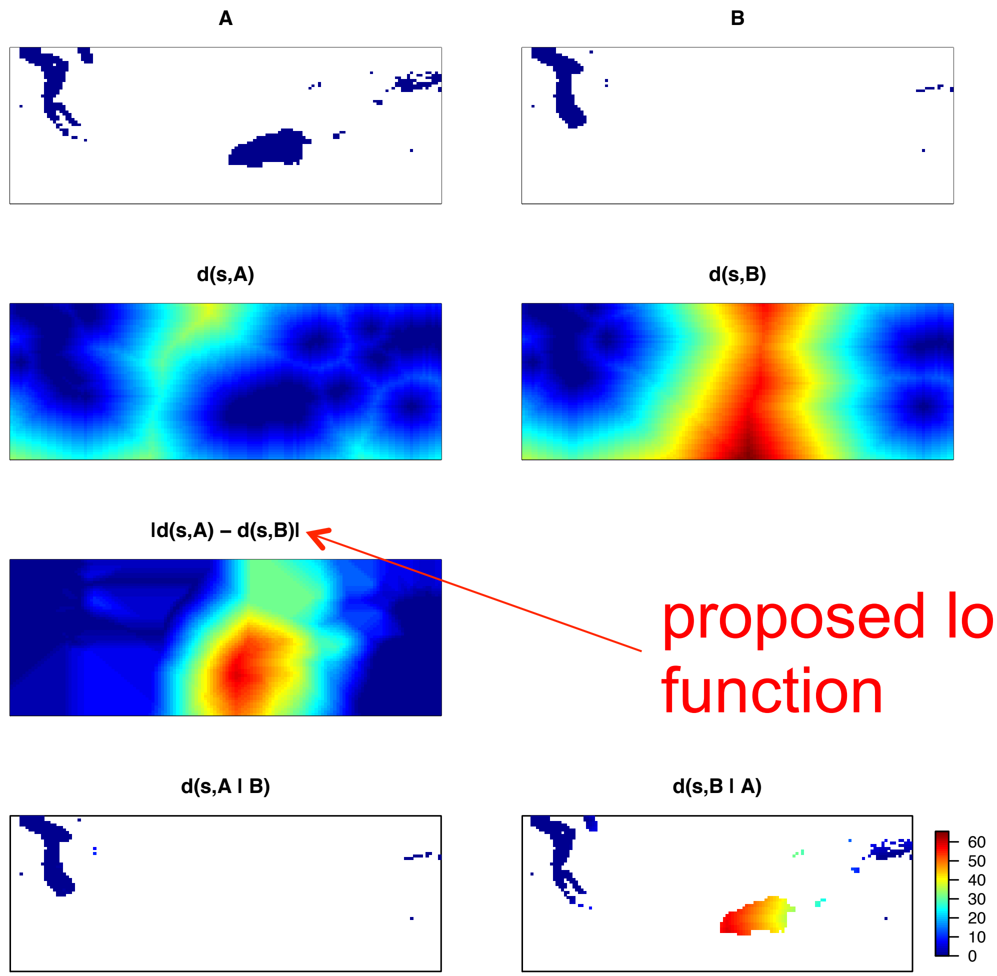
Binary fields obtained via setting all values below 5 mm to zero.

Distance maps can be computed efficiently, and many summary measures are based on them.



proposed loss function

G. (2013, *MWR*, **141** (1), 340 – 355)

Accounting for Location Errors

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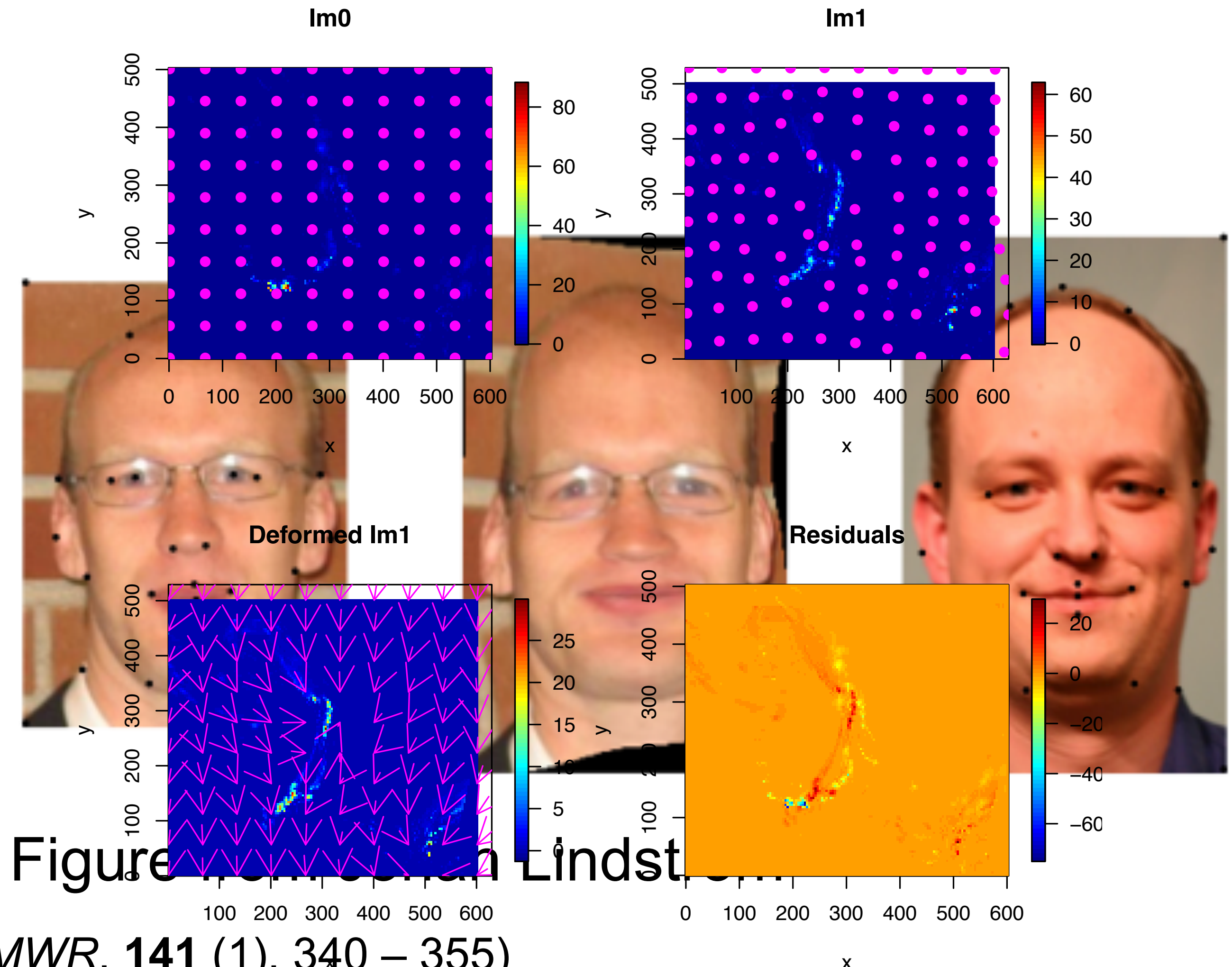

Figure [illegible] Lindst[illegible]
G. (2013, *MWR*, **141** (1), 340 – 355)

Accounting for Location Errors

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Loss function =

Distance from original location of each point to warped location

+

Loss (e.g., square error, absolute error) at each point between observation and warped value

G. (2013, *MWR*, **141** (1), 340 – 355)

Accounting for Location Errors

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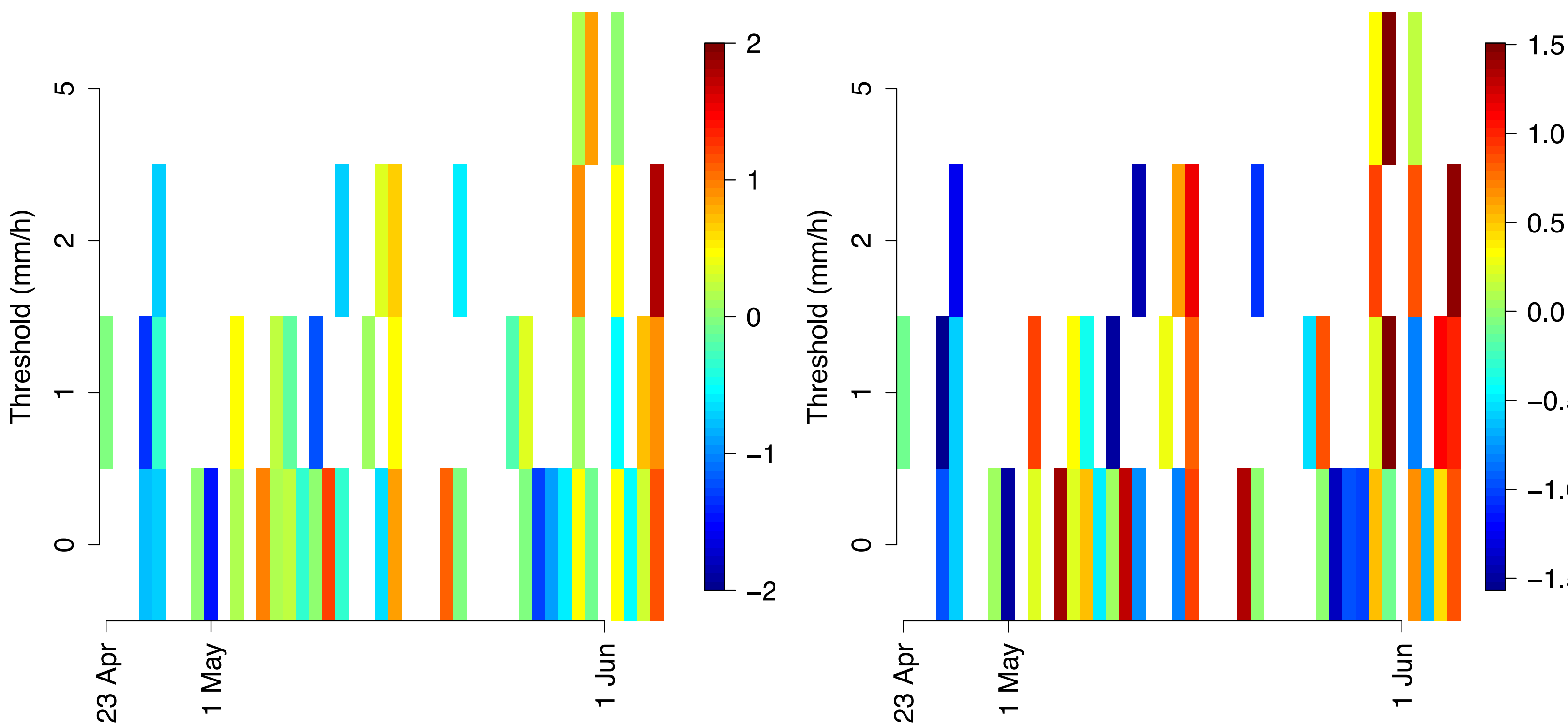
32 test cases (NSSL/SPC Spring 2005 Experiment). ARW-WRF vs NMM

mean(**d**) S



G. (2013, *MWR*, **141** (1), 340 – 355)

Summary of SPCT and SPCT + Warping

NCAR

- hypothesis test (or confidence intervals) for competing forecast models.
- Accounts for spatial correlation.
- Does not require a distributional assumption about the underlying fields (only the test statistic, S).
- Works for any loss function (though some work better than others).
- powerful test
- `R` software package `SpatialVx` conducts the test.
- Image Warping loss allows one to also account for location and small scale errors.

Future Work

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- Compare with variance inflation factor and block bootstrap methods.
- Add image warping to `SpatialVx`, and create an image warping package for R.
- Space-Time Prediction Comparison Test?
  - Challenge is to make simulations with known spatiotemporal correlation structures.
  - Test whether a space-time separable covariance can be used even in the case of non-separability.
  - Is it just overkill?
  - Image warping can be done in space and time together (see, e.g., G. et al., 2010, *NCAR Technical Note,* TN-482+STR).